A

THEORETICAL AND PRACTICAL TREATISE

ON

ASTIGMATISM

BY

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WITH FIFTY-NINE DIAGRAMS AND ILLUSTRATIONS.

TO THE READER.

The addition of one more medical book — even though it be a small one — to the scores that are annually issued from the press, carries with it a demand, on the part of the reading public, for its *raison d'être*.

The cause of the existence of this little work has its foundation in my own needs as developed in my studies of refractive anomalies, and in my conception of the needs of others as manifested to me during the last eight years as a teacher of general and special students and general practitioners.

An examination into the statistics of eye-affections shows that the various anomalies of refraction form about one-third of the whole number of eye cases presenting themselves for treatment in private practice, and of these two-thirds, or about 20% of the whole, suffer from astigmatism in an appreciable degree.

No one, then, I think, will deny to astigmatism the worth of a treatise special to itself, particularly when there is taken into consideration the great difficulty encountered in clearing away the perplexities, uncertainties and paradoxical manifestations which enshroud many cases of the anomaly, and which are the despair of the beginner in refraction studies.

And, while fully aware that Time and Patience are two most important and indispensible factors in unraveling the tangled threads of evidence, it cannot be doubted that nowhere in the whole range of medical practice, is accurate knowledge, based on positive science, of such avail as in the diagnosis of astigmatism. To lead to such accurate knowledge through the paths of positive science has been the chief incentive to my labor.

The preliminary chapter, embracing the fundamental principles of optics which are needed for a clear comprehension of what follows, was introduced because experience shows that

most students of medicine in this country are lacking in such knowledge. No one can more fully realize than I the almost impossible task of giving a concise yet perfectly clear exposition of optics in a single chapter. Those, therefore, to whom my work appears not sufficiently elementary, I would refer to the appendix to Dr. Loring's "Text-book of Ophthalmoscopy," and to a most excellent and remarkably simple and lucid "Treatise on Simple and Compound Ophthalmic Lenses," by Chas. F. Prentice, of New York, where they will find the subject treated of in the simplest form possible, while those desiring a fuller application of these laws will do well to consult the first part of the translation of Dr. Landolt's treatise on the " Refraction and Accommodation of the Eye;" three works published since this treatise has been in type.

One word in regard to the bibliography. Without asking any undue indulgence for its imperfections and shortcomings, we would call to the mind of the captious critic the apothegm of an old and experienced bibliographer: "If a man have a pride of accuracy, and desires to be cured of it, let him make a bibliography."

I trust that our labors in this regard may not be without value, particularly to the future writer on the subject, for we believe that, in that far away time, should the New Zealander, prowling among the ruins of the great medical library on the banks of the Potomac, stumble on a copy of this work, he will find recorded there the title of every important paper on the subject that has appeared up to the year of Grace 1886.

I desire to return publicly my acknowledgment of valuable assistance in the construction of the work — the bibliography in particular — received from my former assistant, Dr. Louis Kolipinski.

<div align="right">SWAN M. BURNETT.</div>

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TABLE OF CONTENTS.

CHAPTER I.

CHAPTER II.

CHAPTER III.

CHAPTER IV.

CHAPTER V.

CHAPTER VI.

CHAPTER VII.

CHAPTER VIII.

CHAPTER IX.

CHAPTER X.

CHAPTER XI.

CHAPTER XII.

CHAPTER XIII.

LIST OF ILLUSTRATIONS AND DIAGRAMS.

CHAPTER I.

Definition of Astigmatism—Fundamental Laws of Optics
Refraction, by spherical surfaces—Formation of Im-
ages by Convex Refracting Surfaces—Test
Glasses and their numbering.

§ 1. Astigmatism is a condition resulting from any irregu-
larity in the refraction of an optical apparatus which renders
impossible the formation of clear and distinct images of ob-
jects in all their parts.

§ 2. In order to satisfactorily study this irregularity in re-
fraction, it will be necessary to first understand those laws
which have been found to govern refraction by surfaces having
a regular, spherical form.

To this end it will be well to call to mind here, at the begin-
ning, some of the elementary principles of optics, since they
form the foundation of all that follows, and will be indispensa-
ble to beginners for their further study of the subject before us.
We should bear in mind that:

a.—From every point of an illuminated object there go out
luminous rays in every direction free to the passage of light.

b.—These rays of light move always in straight lines, even
when thrown out of their original course by reflection or re-
fraction.

c.—Rays of light, while always mathematically divergent,
when they have arrived at a distance of eighteen or twenty
feet from their source, can be, for all practical purposes, con-
sidered *parallel*, and they remain so for an infinite distance.
For the lenses in common use in ophthalmology, and for
the eye itself, therefore, all distances greater than twenty
feet are practically infinite.

The laws governing reflection and refraction are few and

simple.   The law of reflection is that the angle of reflection
is equal to the angle which the incident ray makes with per-
pendicular to the reflecting surface at the point of inci-
dence.

The angle (or amount) of refraction is governed by two con-
ditions, 1) the angle which the incident ray makes with the
perpendicnlar to the refracting surface at the point of inci-
dence, and 2) by the index of refraction (or density) of the re-
fracting medium.

All questions in optics, however complicated, must finally be
brought into harmony with these few general iaws for their
perfect solution.

As in astigmatism we have to do mostly with refraction, we
shall pass by any consideration of the laws of reflection as ap-
plied to optical apparatus.

Fig. 1.
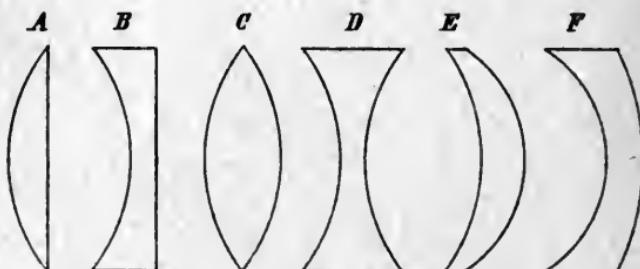
DIFFERENT FORMS OF SPHERICAL LENSES.   *A*, Plano-convex.   *B*, Plano-con-
cave.   *C*, Double-convex.   *D*, Double concave.   *E*, Convex meniscus.   *F*, Concave
meniscus.

§ 3.   The function of every optical appliance is to change
the course of the rays of light falling upon it from a source of
illumination, and, usually, in such a manner, that there shall
be formed an image of some object.

The optical apparatus with which we have to do in refrac-
tion are called *lenses*, because they are usually of a shape
somewhat resembling the seed of a lentil.

Those in common use are divided into two general classes,
called *convex* and *concave*, according to the character of their
curved surfaces.   *A*, in Fig. 1 is a convex lens, *B* is a concave lens.

In these it will be noticed that one surface is curved while the other is straight or plane—they are therefore called *plano-convex* and *plano-concave*, to distinguish them from those which have both surfaces curved as in *C* and *D*, and which are called from this circumstance *double* (or *bi*) *convex* and *double* (or *bi*) *concave* respectively. The forms *E* and *F* are called *meniscuses* because of their fancied resemblance to the moon at its quarter. In meniscuses both surfaces are curved in the same direction. When the *concavity* is predominant it is called a *concave meniscus*, *F*; when the *convex* curve is in excess it is called a *convex meniscus*, *E*.   All *convex* lenses are also designated as *plus* (+), *positive* or *collecting*, and all *concave* lenses as *minus* (−), *negative* or *dispersing*.

Fig. 2.
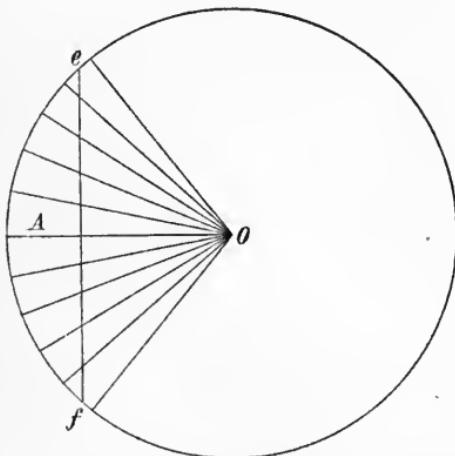
The formation of a Spherical plano-convex lens.

It will be observed that the curved surface in all these lenses is regular, like that of a globe or sphere, and for this reason they are called *spherical*. It is to be remembered that these drawings represent only meridional *sections* of the lenses.   It is characteristic of the spherical lens that the sections made in all its diameters or meridians are similar, and therefore the curvature of its various meridians must be the same.

§ 4.   All lenses of this character, therefore, are sections of a sphere as shown for the plano-convex form in *A* Fig. 2; and

the radii of curvature of the lens are the radii $O\,e$, $O\,f$, etc..
of the circle of which its surface forms a part.

§ 5.—The optical properties of a refracting system depend
upon what are called its *cardinal points*, which are six in num-
ber, namely: two principal foci, two nodal points and two prin-
cipal points, all of which are situated on the principal axis of
the system.

The *first principal focus* is the point where the incident rays
should cross in order that the corresponding emergent rays
shall be parallel to the principal axis. The *second principal
focus* is the point of crossing of the emergent rays when the
incident rays are parallel to the principal axis. These points
are *real* or *virtual* according as the refracting surface is *positive*
or *negative*.

The *principal points* possess the following properties: When
an incident ray, prolonged if necessary, passes through the
first principal point, the corresponding emergent ray, or its
prolongation, passes through the *second principal* point, but
the incident and emergent rays *are not parallel*.

The *nodal points* have this characteristic: that if an incident
ray or its prolongation passes through the *first nodal* point, the
corresponding emergent ray coincides with a straight line par-
allel to the incident ray and directed to the *second nodal point*.

The planes passing through the principal foci are called the
focal planes, and the planes passing through the principal points
are called the *principal planes*. The *principal planes* enjoy this
property: The incident and emergent rays cut the first and
second principal planes in the points situated on the *same side*
and at the same *distance* from the principal axis of the system.

The *first focal distance* is the interval between the first prin-
cipal point and the first principal focus; the *second focal dis-
tance* is the distance from the second principal point to the sec-
ond principal focus.

The simplest form of dioptric system is a curved surface
separating two transparent media having different indices of
refraction. In Fig. 3 $M\,N$ represents such a surface with its
center of curvature at C, through which the principal axis $X\,X'$

passes. On account of its limited amplitude the curve $M N$ can be considered as coinciding with a plane tangent to its surface at A. It is evident that any incident ray, $D I$, and its corresponding refracted ray $I G$ must cut this plane at the *same point* I, therefore the *two principal planes* are *one and the same*, and the *two principal points* through which they pass are one and the

Fig. 3.
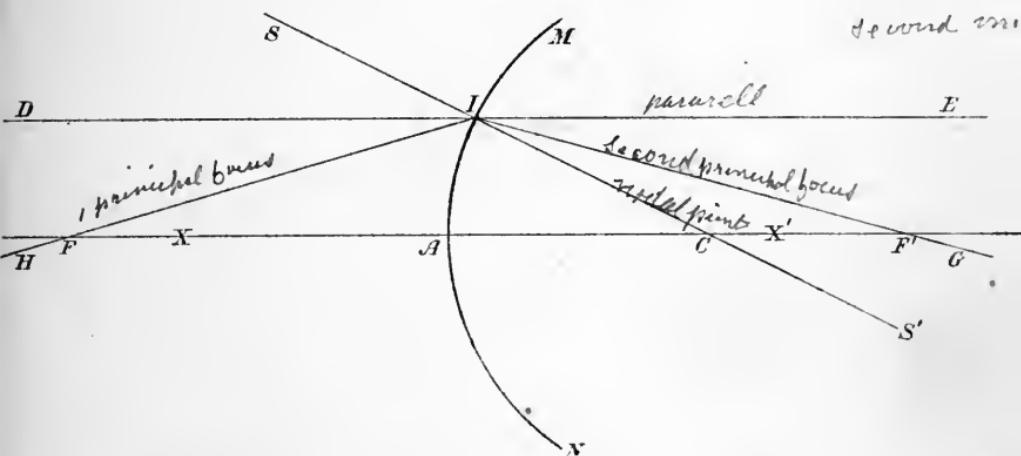
Showing the Refraction of Rays by a simple Convex Surface.

same and must *coincide with the apex A* of the curved surface $M N$.

If we draw from S a line which passes through the center of curvature at C it will be a normal to the surface $M N$ at I, since it coincides with one of the radii of curvature. Any incident ray, therefore, following this direction would, after it entered the second medium, continue without deviation through C towards S'. C therefore performs the office of the *two nodal points*, which are reduced to one, and this coincides with the center of curvature. The *second principal focus* will be at F', because it is the point of crossing of the emergent ray $I G$ when its corresponding incident ray $D I$ is parallel to the principal axis $X X'$. F is the *first principal focus*, because it is the point where the incident rays should cross in order that the corresponding emergent refracted ray $I E$ shall be parallel to the principal

axis. The *focal distance*—that is, the intervals between these focal points and the principal points—will depend on the radius of curvature and the index of refraction of the second medium. The planes passing through the focal points *F* and *F'* are called the *focal planes*.

When the refracting system is a bi-convex lens the re-

Fig. 4.
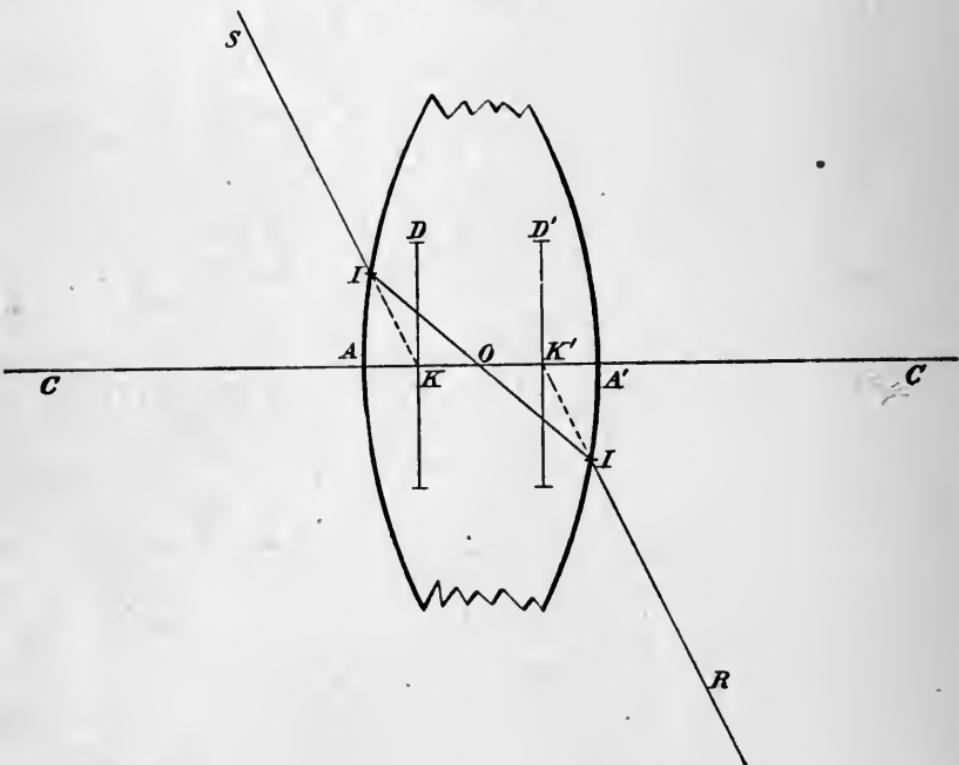
CARDINAL POINTS OF A BI-CONVEX LENS.—*K K''*, the two nodal points. *D D'*, the two principal planes. *O*, the optical center. *C C*, Focal points.

lative positions of the cardinal points are somewhat different. The two *nodal points* are found within the lens at *K K'* as shown in Fig. 4. Let *S* be an incident ray which, after refraction, passes through *O*, which is the optical center of the system. After it emerges as a refracted ray, *I' R*, it will assume a direction parallel to the incident ray *S I*. A prolongation of the emergent ray *R* in the lens would strike the principal axis at

K', and a prolongation of the incident ray S in its original direction would strike the principal axis at K. Thus K and K' fulfill all the requirements of nodal points.

It has been demonstrated that any incident ray striking the plane D at a certain distance from the axis will have a corresponding emergent ray cutting the plane D' at an equal distance from the principal axis. D and D' therefore fulfill all the requirements of *principal planes*, and as they pass through K and K' respectively these points must coincide with the principal points. Therefore the *nodal and principal points are the same.*

Fortunately for the study of lenses placed in air, the six cardinal points can be reduced to two. The two principal and two nodal points being the same, and the two nodal points falling very close together they can, for glasses in ordinary use, be considered as coinciding with the second nodal point; and where the medium is the same on both sides of the lens the focal distance is the same for the two sides, so that we have to do really with only the one nodal point and one focal distance.

The focal distance of a lens, which is but the expression of its *refracting power*, is governed, as stated in the previous paragraph, by its radius of curvature and the index of refraction of the material of which it is composed. It has been found that when the glass, of which a lens is made, has a certain index of refraction[1] (1.5) the focal distance is just double the radius of curvature. When, therefore, the lens is a double convex or concave, its focal distance is equal to its radius of curvature.

§ 6. The action of convex and concave lenses on light is of an opposite character. Rays after passing through a convex lens tend to come together at a point in front of the lens, while rays after passing through a concave lens diverge, as though they came from a point behind the lens.

It is the office of all collecting systems (of which all

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[1] It is commonly believed, and many very good authorities have fallen into the error, that flint glass is harder than crown glass. The index of refraction of flint glass is higher than that of crown glass, but the lead entering into its composition makes it softer.

convex lenses and the eye are representatives) to form at their
foci small and inverted images of extraneous objects.

By the aid of the few laws, which we have laid down in the pre-
ceding sections, it is possible to show by construction, and with-
out the help of any mathematical formulæ, how an image of
an object is formed by a convex refracting surface.

We have endeavored to do this in the construction of Fig. 5.

Let MN represent a curved surface separating two transpar-
ent media of different densities (such as air and water), and for
the sake of simplicity of construction and demonstration we
will assume that the refracting power of the water is twice

Fig. 5.
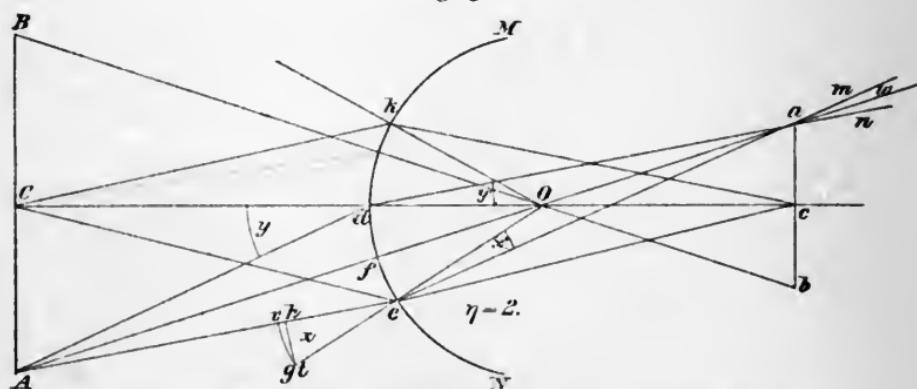
Showing how an Image is formed by a Convex Refracting Surface.

that of air—that is the index of refraction $(\eta) = 2$. The center
of the curved surface is O, and Of, Oe, Od, and every line
drawn from the surface through O are its radii. AB is an
object from all points of which rays proceed in every direction
towards MN.

Now, in order to have an image of an object, the rays coming
from every point of the object must be again brought to
another point, and it is the function of the curved surface to do
this.

Let us take a bundle of rays proceeding from the point A.
Of these one will fall on the surface MN at e, one at f and one
at d. We will now follow these rays after their passage into the

second medium.   The ray *A f* being perpendicular to the sur-
face and passing through the center *O* (which is the *nodal point*
of the refracting surface) being, in fact, but a prolongation of
the radius *O f*, suffers no refraction, but passes straight on in
the direction of *w*.   The ray *A e*, however, falls upon the curve
M N obliquely, and makes an angle with the perpendicular *g O*
to the surface at the point of incidence *e*.   This perpendicular
is nothing more than a prolongation of the radius *O e*.   As
stated in § 2, the amount of refraction which a ray undergoes
depends upon the angle it makes with the perpendicular to the
surface, and the difference in the refracting power of the media
(index of refraction).   Observation and experiment have shown
that when a ray passes from a rarer to a denser medium, it is
drawn towards the perpendicular, and in exact ratio to the
difference in their refractive powers as indicated by the
index of refraction.   This difference is expressed naturally by
the difference in the size of the angles made by the incident and
refracted rays with the perpendicular.   The size of an angle is
usually expressed mathematically by its *sine*, and the index of
refraction is said to be equal to the difference of the sines.

In constructing such a diagram as Fig. 5 it is easy to meas-
ure the size of these angles.   The angle of incidence *A e g*,
for example, may be measured directly by means of a gon-
iometer, or with *e* as a center we can describe the arc *t v*, and
with the same radius the arc *x'* on the perpendicular *g O*.   The
line *t h* let fall on the perpendicular from the incident ray is the
sine of the angle *x*, and the *sine* of the angle *x'* must be
just one-half as large—that is the length of the line at *x'*
must be half the length of *t h*.   A line *e m* drawn through *e*
and the extremity of the line at *x'* must then be the course of
the ray *A e* after refraction, and it will cross the line *a b* at *a*.
The course after refraction of another ray, *A d*, can be deter-
mined by measuring the sines of the angles *y* and *y'* in the same
manner, and it will be found that, if *M N* is a regular curve it
will pursue after refraction the course *d n* and cross the other
two rays *e m* and *f w* at *a* also ; *a* will therefore be the *focus* of
all the three rays.

By a similar method of construction it can be shown that all the rays going out from A and falling on the curve $M N$ between d and e, and in fact anywhere on its surface will likewise be brought together at a. The point a must therefore be the image of the point A in the object $A B$.

By the same method of construction it can be shown that all rays coming from the point C will be brought to a focus at c, which will be the image of C. By the same law all rays emanating from B will be united at b forming there an image of that point ; and so for every point between A and B there will be a corresponding focus and image between a and b. $a b$ will therefore be the image of $A B$. It will be observed that it is *inverted* and *smaller than the object.*

$M N$ represents only one meridian of a curved surface and $A B$ only the section of a plane surface, but it is evident that if $M N$ were a spherical surface curved equally in all its meridians a small and inverted image of an object on the *plane $A B$* will be formed at $a b$.

It will be observed that with the exception of the curve $M N$ all the lines in Fig. 5 are straight, and this in surfaces of very limited amplitude can also be practically considered as a straight line. These lines in this figure go to the construction of a large number of triangles, and since we can always know the size of the object and its distance from the refracting surface, and the index of refraction of the second medium, we can, by the application of the few simple rules of plane trigonometry, easily find the position and size of the image and its distance from the refracting surface.[1]

§ 7.—In ophthalmic practice a number of lenses of different refracting powers are used for the purpose of determining the refraction of the eye and for correcting optical anomalies. This series of lenses used for examining the refraction are called trial lenses. These sets are constructed with a view to having all the glasses that are necessary with none that are

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[1] The simplest exposition of the elements of optics with which I am acquainted is the little volume by Prof. Gavarret of Paris on " Images par réflexion et refraction:" No translation has been published in English.

superfluous.   In practice we seldom have use for a lens with a
focus shorter than two inches,—for even an aphakial eye does
not often need a lens stronger than two or two and one-half inch-
es focus,—or longer than 160 inches, since rays become practi-
cally parallel at the distance of 240 inches. The majority of trial
cases in use are composed of lenses with various foci embrac-
ing these two extremes.   Most of them have thirty pairs of
convex and the same number of concave lenses.

It is advantageous to have this series with as nearly as possi-
ble regular intervals between adjoining numbers.

§ 8.   There are two methods of numbering these lenses, one
the inch, or old; the other the metric, or new.   The advant-
age of the inch system is that it gives directly the focal dis-
tance of the lens; one of its disadvantages is that the power
of the lens must be expressed in vulgar fractions.   As the
standard of refracting power is a lens with a focal distance of
one inch ($^1/_1$), and as the refracting power of a lens is in inverse
ratio to its focal distance, all lenses with foci longer than one
inch must be expressed in fractions; thus a lens of twelve inches
focus has a refracting power of only one-twelfth; one of eight-
teen inches, one eighteenth, etc., etc.; whereas one of one-half
inch focal distance would have a refracting power of two.
There are few people who can add and subtract vulgar fractions
without resorting to pencil and paper, and this is a great incon-
venience in the combinations of lenses which we sometimes
find it advantageous to make rapidly in practice.   Another dis-
advantage is, that the inch has not the same length in all
countries; so that a one twentieth in Prussia or Switzerland
would not be the same as one-twentieth in England and Amer-
ica.   It is very desirable, therefore, to have a universal uniform
numbering of lenses, whose power shall be expressed in whole
numbers, or decimals, so that they can be easily added and
subtracted.

§ 9.   This we have in the *metric system*.   Here the standard
refracting power is a lens with a focal distance of one meter,
and as it is the measure of refraction it has been called a
Dioptry (D).   A lens having twice the refracting power, with

a focal distance of one-half a meter (50 cm.) is numbered 2 D; one with three times the refracting power but one-third the focal distance (33 cm.) is No. 3; while one with one-half the refracting power but double the focal distance (two meters) is called 0.50 D; one with one-quarter the refracting power, but four times the focal distance (four meters), is 0.25 D.

It is easy, however, to convert one system into the other. The meter is about forty English inches (39.37), so it is only necessary to divide forty by the number of dioptries in order to have a close approximation to the corresponding focal distance in inches, thus: 2 D = $^{40}/_2$ = 20 inches; 4 D = $^{40}/_4$ = 10 inches; 5 D = $^{40}/_5$ = 8 inches, etc., etc. On the other hand. if you have the focal distance in inches, it is easy to find the number of the corresponding D, by dividing forty by the number; thus 10 inches = $^{40}/_{10}$ = 4 D, 12 inches = $^{40}/_{12}$ = 3.33 D, 5 inch = $^{40}/_5$ = 8 D, 10 inch = $^{40}/_{10}$ = 4 D.

The following table comprises the number of glasses usually found in trial cases expressed in dioptries, with their focal distances given both in millimeters and inches.

TABLE I.

Number of Dioptries.	Focal Distance in Millimeters.	Focal Distance in English Inches.	Number of Dioptries.	Focal Distance in Millimeters.	Focal Distance in English Inches.
0.25	4000	158	5.5	182	7.18
0.50	2000	79	6	166	6.6
0.75	1333	52.3	7	143	5.64
1	1000	39.5	8	125	4.9
1.25	800	31.6	9	111	4.4
1.50	666	26.3	10	100	3.9
1.75	571	22.5	11	91	3.6
2	500	19.7	12	83	3.3
2.25	444	17.5	13	77	3
2.50	400	15.8	14	71	2.8
2.75	363	14.33	15	67	2.6
3	333	13.16	16	62	2.5
3.5	286	11.2	17	59	2.3
4	250	9.9	18	55	2.2
4.5	222	8.8	20	50	1.9
5	200	7.9			

In the following pages, whenever we shall have occasion to designate the power of lenses we shall, in order to give practice to the beginner in the use of the two systems, employ them indiscriminately; the inch system being always expressed in vulgar fractions, and the metric system in whole numbers and decimals.

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CHAPTER II.

Refraction by Ellipses, Spheroids and Ellipsoids—Focal Interval of Sturm—Character of the Focal Lines—Cylindrical Glasses.

§ 10. When we come to deal with a surface departing, even to a limited extent, from a spherical form, the few simple rules of refraction laid down in the preceding chapter no longer apply, and there is no one point in the image where all the rays coming from any single point of the object meet. From this circumstance such a surface is called *Astigmatic* (from *a*, negative prefix, and στιγμα, a point.)

§ 11. But there are some surfaces deviating from the strictly spherical form for whose refraction some rules can be formulated. These are the *ellipsoids* (including the paraboloids and hyperboloids). This is possible only because the outlines of ellipses follow a regular course in their conformation. All other deviations from the strictly spherical form are *irregular* in outline, and refraction by such figures is governed by no rules that can be applied to a class of cases.

§ 12. For this reason ASTIGMATISM is divided into two distinct forms: *regular* and *irregular*.

§ 13. There is one form of *regular astigmatic surface* where the curve, instead of representing the section of a sphere, is the section of a *spheroid*, a figure formed by the rotation of an ellipse around one of its axes. In such a figure, every section parallel to the axis of rotation is an ellipse. All sections of such a figure parallel to the other axis and at right angles to the axis of revolution are circles.

§ 14. Since the total refraction of a spheroid is represented by the sum of the ellipses into which it may be divided, it is of

fundamental importance that we study in some detail the optical properties of ellipses.

Geometrically, an *ellipse* is " a plane curve traced by a point which moves in such a manner that the *sum* of the distances from the fixed points is always the same. The two fixed points are called the *foci* of the ellipse."(*Loomis' Analyt. Geom., 2d ed., p. 103.*)

Fig. 6.
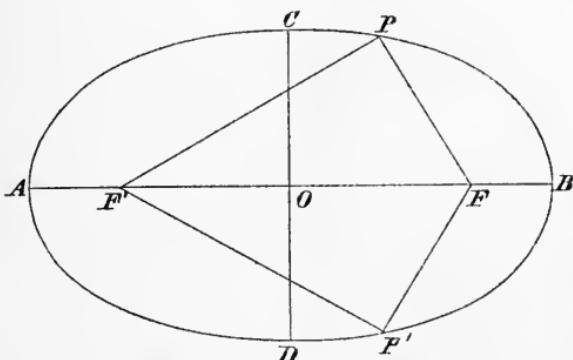
AN ELLIPSE.—*A B*, its Major Axis. *C D*, its Minor Axis. *F' F*, its Foci.

Fig. 6 represents such a figure. *A B* is the diameter or *major axis*, and it is characteristic of it that it passes through the *foci F* and *F'* and through the *center* of the figure *O*, which is the middle point in the straight line, *A B*, uniting the two foci. The conjugate or *minor axis*, *C D*, is the diameter perpendicular to the major axis *A B* at the center *O*.

The curve *A C P B P' D* is elliptical because the sum of the distances of its every point from *F* and *F'* is the same. No matter at what place on its curve *P* or *P'* are found, the sums of *P F'* and *P F*, and *P' F* and *P'F'* are always the same.

It is evident at a glance that refraction by such a surface must differ from that by a sphere, since in a sphere the radius of curvature, one of the factors on which the focus of the refracting surface depends, is the same for all parts of the curve, while in an ellipse it changes at each successive point.

In considering refraction by an elliptical surface we shall

have two separate forms to deal with: one in which the light
falls on the sharper end of the ellipse and in the direction of
the major axis *A B*; and the other where it falls on the blunter
end, and in the direction of the minor axis *C D*.

We have given in § 6, the method for finding the direction
of a ray after refraction by a spherical surface and by apply-
ing the same laws here we can find the direction of any single
ray after its refraction by the surface of an ellipse. We
have for that purpose to know only two things, viz: the index
of refraction of the refracting medium, and the angle the inci-
dent ray makes with the perpendicular to the surface at the
point of incidence. The second of these data we could easily
get in the sphere, for we have only to prolong the radius of
curvature to get the normal to the curved surface at any given
point. The angle is then measured as explained in § 6.
But in an ellipse it is not so easy to get the normal at any
given point, because there is no one center of curvature from
which radii can be drawn. But there is a well-known theorem
which enables us to do it quite readily.[1]

According to this theorem, *all circles and ellipses whose dia-
meters and major axes correspond have the same subtangents.*
We have constructed Fig. 7, which represents the sharper end
of the ellipse in accordance with this theorem. Let *a b* repre-
sent the major axis of the ellipse of which *b p* is a portion;
from *a* as a center and *a b* as a radius, draw the segment *r s* of
a circle. Let *v* and *y* be the rays parallel to the axis and inci-
dent at *k* and *i*. Through the points *k* and *i* draw *l c* and *n d*
perpendicular to *a b*. These will cut the circle at *q* and *φ*, and
the normals at these points coincide with lines drawn through
them and the center *a* ; and the lines *x u* and *j t* drawn at right
angles to these normals will be tangents at the points *q, φ*. Now,
if the circle and ellipse have the same subtangents *b u* and *b t*,

---

[1]To be found in any treatise on analytical geometry. Compare Loomis' "Ele-
ments of Analytical Geometry," 1873, page 113. "Since the subtangent is inde-
pendent of the minor axis, it is the same for all ellipses which have the same major
axis; and, since the circle on the major axis may be considered as one of these ellip-
ses, the subtangent is the same for an ellipse and its circumscribing circle."

then the lines $z\,u$ and $w\,t$, drawn through u and k and t and i must be the tangents to the ellipse at the points k and i, and the lines $m\,f$ and $o\,e$, drawn perpendicular to them, must be normals to the surface at the points of incidence, k and i. We have now all the requisite data, and have only to apply the law of sines, as given in § 6, in order to find the course of the rays

Fig. 7.
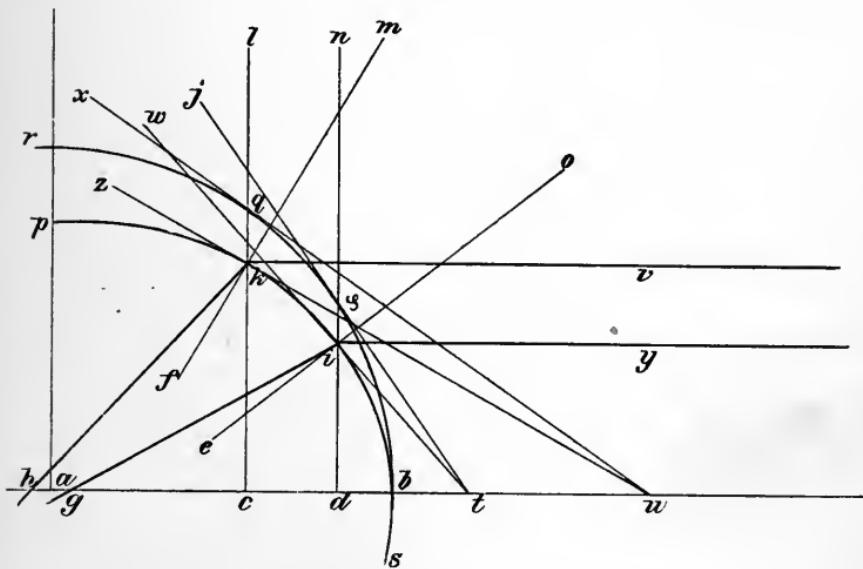
Refraction by the sharper End of an Ellipse.

ig and kh. In this case, in order to have the diagram fall within reasonable limits, we have assumed a refracting index $=$ 3.

It will be seen at a glance that in the case where the rays fall parallel to the long axis of the ellipse, the ray iy, nearer the axis, crosses the principal axis, au, after refraction, *in front* of the more peripheral ray kv,—that is to say, *we have an aberration the opposite in kind to that of an ordinary spherical surface.*

If, however, the light falls on the ellipse in the direction of the short axis, or on the blunter end, as we have it represented in Fig. 8 (which has been constructed according to the same plan as Fig. 7), we find that the more peripherally refracted ray kp crosses the principal axis in front of the more centrally re-

fracted ray *ih*; in other words, we have an *excess of the ordinary spherical aberration.*

It therefore becomes evident,*that if we take a series of curves passing over from the flatter to the sharper end of an ellipse, we will have in the refraction, first, an exaggeration of the spherical*

Fig. 8.
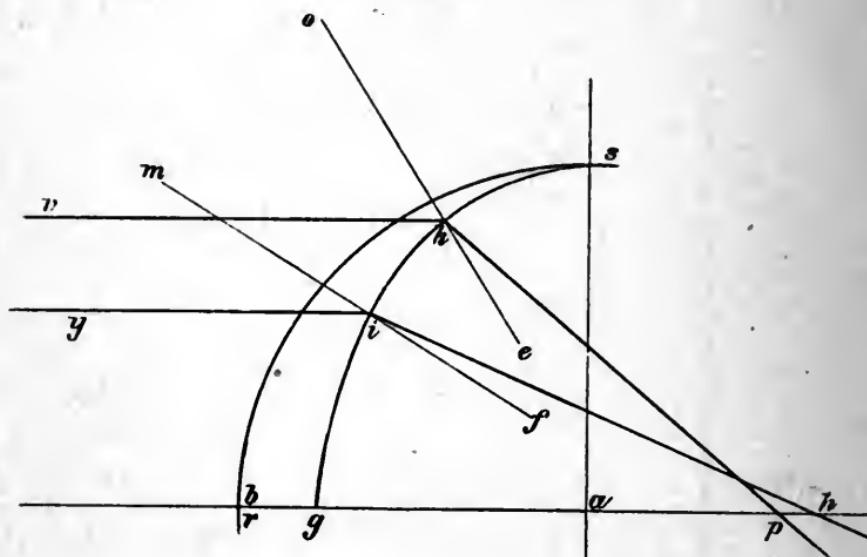
Refraction by the blunter End of an Ellipse.

aberration—which will be greater in proportion to the difference in the length of the major and minor axes—diminishing until the curve becomes a circle, when there will be only the ordinary amount of spherical aberration ; then, as the minor axis becomes shorter, this aberration will still further diminish until it becomes, for any chosen rays, practically zero. As the minor axis still further shortens, the aberration passes over to an opposite kind, and the more central rays cross the principal axis in front of the more peripheral, and this will increase PARI PASSU *with the shortening of the minor axis.*

It follows from this demonstration that deviation from a spherical form does not necessarily involve a lack of focus for some of the rays, and that there is *one* form of ellipse in which monochromatic aberration is practically abolished.

In every other form, however, there is a failure to focus all the rays in one point, and this monochromatic aberration increases with the difference between the major and minor axes of the ellipse. The foci of corresponding rays falling at equal distances from the principle axis do not all come together forming a point, but form a *line* on the principal axis whose length will be in direct ratio to the difference between the major and minor axes.

It is apparent, therefore, that in all figures formed by the revolution of an ellipse about one of the axes, whether in the form of oblate or prolate spheroids, there will be, with the exception of one particular case, a monochromatic aberration such

Fig. 9.
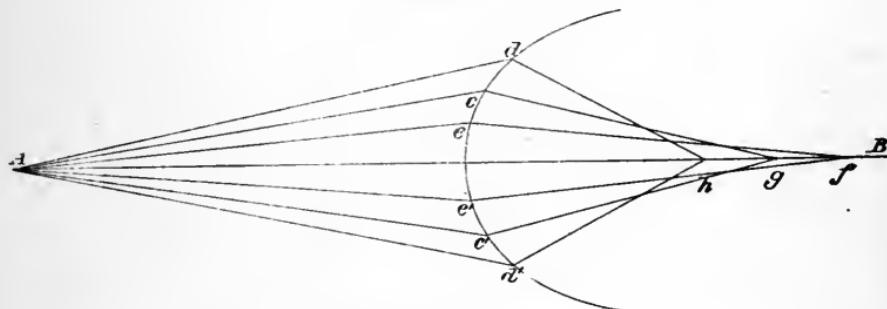
Refraction by a Spheroid.

as to prevent the formation of clear and distinct images on any single *focal plane.* All rays, however, as we have seen, proceeding from a point on the axis will be united after refraction at some place on that axis, and those which fall on the refracting surface at equal distances on either side of the axis at the *same* point, since the radii of curvature are the same for all points equidistant from the axis of revolution. The rays *A d* and *A d'* in Fig. 9 will be united, in accordance with the law of refraction by ellipses, when the light falls on the blunter end as demonstrated above, at *h*; the rays *A c* and *A c'* at *g*, and *A e* and *A e'* at *f*, and so on for the whole bundle of rays coming from *A* and falling on the spheroidal surface *d d'*. The position of the foci will be in an inverse order when *d d'* is the sharper end of the ellipsoid. If there were an object at *A*,

therefore, there could not be a distinct image of all the points of its surface on any one plane perpendicular to AB since the rays coming from each individual point would have their foci on the different planes between f and h, according to the position of their points of incidence on $d\,d'$. In the place, therefore, of a focal point there is a *focal interval f h*, which is measured by the distance between the focus of the points of least and greatest refraction on the spheroid. It is characteristic of refraction by a spheroid that while refraction in all the meridians is the same, it is not the same at all points of the same meridian.

Such a form of refracting surface has its nearest representative in the eye in certain forms of conical cornea, but as this condition is rarely met with in a typical form, being nearly always associated with other irregularities in refraction, we will defer its consideration in detail until we treat of *irregular astigmatism*.

§ 15. Those spheroids formed by the revolution of an ellipse about one of its axes are sometimes called *bi-axial ellipsoids* in order to distinguish them from another form of ellipsoid called *tri-axial*. The latter is also sometimes called a *compressed spheroid*, because if a spheroid be compressed in the direction of its minor axis so as to shorten it in that meridian, we would have a figure with a major axis and two minor axes, all of different lengths. If we make a section of the base of such a figure, we have, instead of a circle as in the spheroid, an ellipse, as shown in Fig. 6, in which AB would be the *long minor axis*, and CD the *short minor axis*, the major axis passing through O and the apex of the figure. Moreover, it would follow that there would be a meridian of greatest curvature which would correspond to the shorter minor axis, and a meridian of least curvature which would correspond to the longer minor axis. It is further apparent that *these two meridians must be at right angles to each other.*

Refraction by an ellipsoid with such an irregular surface is much more complicated than that by a sphere or spheroid, and it is impossible to formulate any laws in regard to it that will apply to the surface as a whole.

In the spheroid, the minor axis being the same for all the meridians of the surface, all rays falling at equal distances from the principal axis are brought together at the same point on the optical axis, as shown in § 14 (Fig. 9). When, however, the minor axis is different for the two principal meridians, as in Fig. 6, this can no longer be the case, and only those rays falling in the principal meridians are united on the optical axis, *and even these will not all be at the same point.* The rays falling in the other meridians are scattered, and when meeting at all, cross at some point off the optical axis.

Fig. 10.
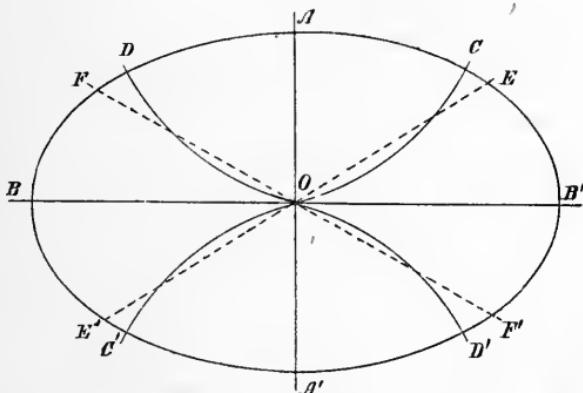
LINES FORMED BY THE NORMALS TO A TRI-AXIAL ELLIPSOID. $A A'$ and $B B'$ are the principal meridians. The curved lines $C C'$ and $D D'$ represent the position of the normals in two intermediate meridians.[1]

This scattering of the rays is due to the peculiar "skew" form of the refracting surface, which changes its curvature at each successive point, so that normals to the surface, (which, as we have seen in § 4, control the direction of the refracted rays) fall in the same plane only in the two principal meridians

[1] This condition can be very effectively shown on a model of a triaxial ellipsoid made in wax. Pins are stuck in rows corresponding to the various meridians, each pin being perpendicular to the surface and representing a normal at that point. It will be seen on an inspection of these rows, that none except those corresponding to the principal meridians form straight lines. All the others are curved, no two points lying in the same plane passing through the apex of the figure. Fig. 10 represents in $C C'$ and $D D'$ the curve of two intermediate meridians, projected on a plane surface. Dr. Knapp was the first, I believe, to construct such a model.

as shown at $A\,A'$, and $B\,B'$, Fig. 10. Normals to all the other meridians do not fall in the same plane forming right lines like $E\,E'$ and $F\,F'$, but each one falling in a different plane make a line curved as in $C\,C'$ and $D\,D'$.

Rays falling in these meridians, therefore, can not be brought to a focus at the same place, since the refracted rays will no longer lie in the same plane as before their incidence. Some will cross above and some below the optical axis, but they can never meet. Only those rays falling in the principal meridians and in the planes parallel to them can be united after refraction, because it is only those rays which lie in the same plane before and after refraction.

§ 16. It is evident, then, that while the rays refracted by such a surface can never meet, at a single point there is, nevertheless, a certain amount of focussing and this will be found on *lines* at right angles to the corresponding principal meridians, and the planes parallel to them, by which the rays are refracted. As there are two principal meridians, the focus of a triaxial ellipsoid will therefore not be a point, but two *lines* perpendicular to each other; one being the focal line of the more strongly curved meridian, lying nearest the refracting surface and perpendicular to its corresponding meridian; the other, the focal line of the meridian of least curvature, being farthest from the refracting surface and perpendicular to its corresponding meridian. The distance between these two lines is called the *focal interval* of *Sturm*, in honor of his having first described it.

In Fig. 13, § 18, $v\,v'$ is the focal line of the meridian of greatest curvature $V\,V'$, and $h\,h'$ is the focal line of the meridian of least curvature $H\,H'$; the distance $m\,n$ between them is the *focal interval*.

Preliminary to a further consideration of the nature of this focal interval we will study briefly the character of the *focal lines* which form its boundaries.

It is common with writers on astigmatism to speak of the anterior and posterior focal *planes*, meaning by these the planes passing through the foci of the principal meridians. To the use of these terms no exceptions can be taken when they are lim

ited in their application to the planes passing through the points mentioned. When, however, the focal planes are considered, as they undoubtedly are by many, in the sense of planes passing through the focal *lines*, the terms are misapplied.

I think the error has arisen from experimental attempts to demonstrate the character of refraction by an asymmetrical system. One of the simplest as well as the commonest of these methods is the combination of a cylindrical and a spherical lens to be described in § 17. This, indeed, makes an astigmatic system, inasmuch as there is no place where all the rays emanating from any point are united after refraction, and in such experiments the anterior and posterior focal lines do correspond approximately with the planes passing through the foci of the meridians of the greatest and least refraction. This is so because the two principal meridians are regularly curved surfaces —sections of a sphere—subject to only the ordinary amount of monochromatic aberration.

If the cylindrical lens acted alone we should have a series of foci on a right line parallel to the axis of the cylinder, as shown in Fig. 15, § 21. Every set of parallel rays $a\,a'$, $c\,c'$, etc., are united after refraction, each in its own plane, which is perpendicular to the axis of the cylinder f, at the points a'', c'', etc., on the line $F\,F'$ parallel to the axis f. The rays passing through the cylinder parallel to f would, of course, suffer no refraction. When, however, a positive spherical lens is added to this cylinder, it has the effect of uniting those rays passing through the cylinder in planes parallel to its axis f, and of advancing the focal line $F\,F'$, forming the focal interval of Sturm, bounded by the anterior and posterior focal *lines*. In this instance these lines are approximately *straight*, because the surfaces of both the cylindrical and spherical lenses are regularly curved, and the difference between the refraction of the central and peripheral portions is expressed by the usual amount of spherical aberration. That is to say, there would be a slight curving of the focal lines, their concavity being toward the refracting surfaces.

The conditions are not, however, the same when we come to deal with an ellipsoid of three unequal axes. Here each of the

meridians is not the section of a sphere, but an ellipse which changes its curve at each successive point. No lenses with such surfaces, so far as my knowledge extends, have been used in making these experiments.

It is very easy, however, to picture in the mind's eye the conditions we should have in refraction by such a surface. We

Fig. 11.
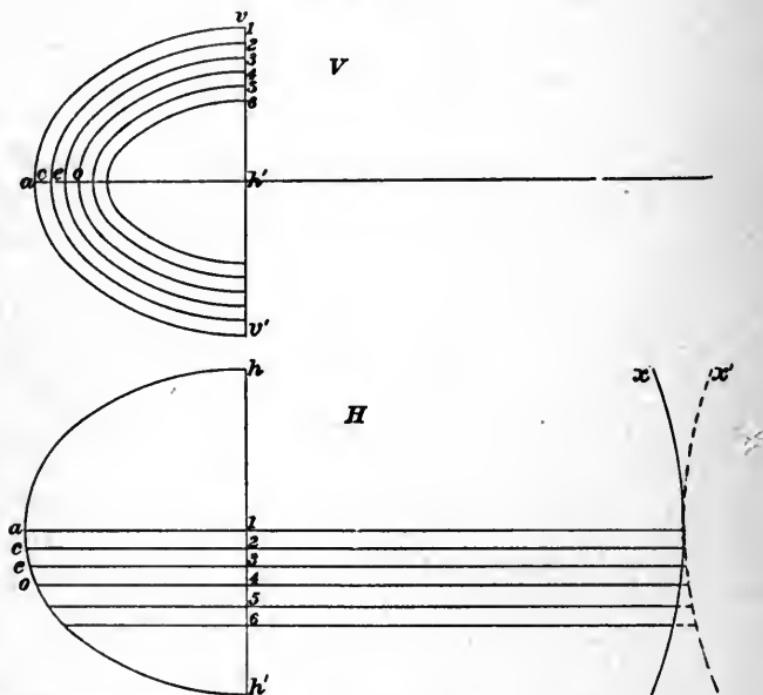
Showing the character of the Focal Lines in refraction by a Tri-axial Ellipsoid.

can imagine the ellipsoid cut into a series of adjacent planes parallel to one of the principal meridians. Each of these planes will then represent, as does the principal meridian to which it is parallel, an ellipse. This is shown in *V*, Fig. 11, where the ellipsoid is divided into a series—1, 2, 3, 4, 5, 6, etc. —of ellipses parallel to the principal vertical meridian *v a v'*.

It must be evident, when we consider the form and the relation of these ellipses to each other, that there will have to be a

wonderful combination of happy circumstances in order to have the foci of rays refracted by all to lie in the same vertical plane.

The foci will all lie in the same *horizontal* plane because the apices of the ellipses, *c, e, o,* (which are the principal points of the refracting ellipses,) are all found in the same plane *a h'*, with the apex *a* of the ellipse 1, which is the principal meridian to which they are parallel; and since all the cardinal points must lie in the same plane, the foci will be found somewhere on the prolongation of the horizontal plane passing through *a h'*.

But it is also very evident from this construction that the principal points can not lie in the same *vertical* plane, for the apices, *a, c, e, o,* of the several ellipses, 1, 2, 3, etc., form part of a curved line which constitutes the ellipse representing the horizontal principal meridian, *h h'*, at right angles to the principal vertical meridian to which they are parallel, as shown in *H*, Fig. 11.

Now, the position of the focus of any one of these ellipses in relation to the focus of the principal meridian depends upon two things; first, upon the radius of its curvature as compared to that of the principal meridian; and, secondly, upon the position of its principal point (from which its focal distance is measured) as compared with that of the principal meridian. These relations, again, depend upon the relations which exist between the three axes of the ellipsoid. Thus, the curve of the horizontal meridian *h h'* in *H*, Fig. 11, formed by the principal points, *a, c, e, o,* etc., of the vertical ellipses, will depend upon the relative lengths of the antero-posterior and horizontal axes, whereas the curvature of the ellipses themselves, on which their foci depend, will be governed by the relation between the antero-posterior and the vertical axes.

Let us take, as an example to illustrate our meaning, the case where rays fall on the sharper end of the ellipsoid, or in the direction of the major axis, and let us assume that the vertical meridian is the more strongly curved, and that the ellipsoid is divided into a series of ellipses parallel to the vertical meridian. It is apparent that these ellipses become constantly smaller, with shorter radii of curvature, as they pass toward the

periphery from the principal meridian, for they finally disappear as a point at the apex h' of the ellipse on the blunter end of the ellipsoid. The effect of this would, of course, be a constant shortening of the focal distance. But at the same time there is a constant recession of the principal points, c, e, o, from the principal plane of the principal vertical meridian passing through a, with, of course, a concomitant recession of the foci.

Now, if we can have such a nice adjustment of the three axes, that these two conditions shall neutralize each other, the line formed by the focal points of the series of ellipses will be a right line, falling in a vertical plane passing through the principal focus of the principal vertical meridian. There is, therefore, *one* possible form of ellipsoid, and only one, in which the focal line of one of its meridians will be a straight line and lie in the plane passing through the focus of the vertical meridian. Whenever there is any deviation from this form of ellipsoid, the focal line will no longer be straight, but curved, and the direction of its curvature will depend on the predominating influence of the one or the other of the above-named factors. If the relation between the major and the horizontal axis is such as to cause the setting back of the principal points to be the more powerful, then the curve would be backward, as shown at x', Fig. 11; should the relation of the axes be such that the shortening of the foci would be in excess, then the curve would be in the opposite direction, or forward, as shown at x. It follows, also, from what has been demonstrated, that in no triaxial ellipsoid can such a relation between the axes exist as to cause *both* the focal lines to be straight; for, as we have seen, there is only one relation of the ellipses that can bring about such a result, and as from the very nature of the figure the ellipses in the two meridians must be different, if one has this form the other cannot have it.

It is not possible to obtain any general formula which would apply to all forms of ellipsoids. It will be necessary to treat every form (of which there is an almost infinite number) separately, and as the task is a tedious and by no means an enviable one, it would hardly be worth while to undertake it unless for some special case.

§ 17. The focal interval of Sturm, as stated in § 16, is the space bounded by the anterior and posterior focal lines which we have just considered as formed by the foci of the meridians of greatest and least refraction.

The direction taken by the refracted rays in their passage through this interval is, owing to the conformation of the refracting surface peculiar and erratic, but it has been analyzed sufficiently to enable us to unravel some of its complications and to understand the general character of figure.

Its general features are made manifest by means of the experiment with a combination of spherical and cylindrical lenses, alluded to in § 16.   Place in front of the milk-glass shade of

Fig. 12.
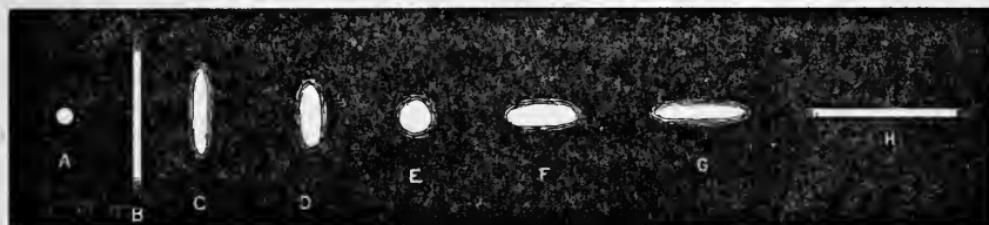
Showing the sections of Sturm's Interval at its various parts.

a lamp a diaphragm with a perforation 1 mm. in diameter. This will serve as a point of light, an image of which is formed on a movable screen, by means of a +4 or +5 spherical lens. This image will be another round point of light as at *A*, Fig. 12.   Now place in front of the spherical a +1 cylindrical lens, axis horizontal.   The round point of light will be immediately changed into a sharp vertical line (*B*) with ill-defined ends. This line is formed by the rays which pass through the spherical lens in the horizontal meridian unaffected by the cylinder. Those that pass through the cylinder in the vertical meridian cross and are scattered before they reach the screen.   Advancing the screen towards the combination of lenses, we have first a vertical oval (*C*), then a shorter but broader oval (*D*), and then a circle (*E*), all of which have ill defined edges, for at none of these points are any of the rays properly focussed.   Advanc-

ing the screen still further we have a horizontal oval (F), which lengthens out at G, and, finally, when the screen is in the focus of the vertical meridian of the system we have again a line (H) horizontal, because it is formed by the rays passing through the vertical meridian.

Astigmatic refraction can also be shown by models made of thread, the first of which was constructed by Knapp, and by the direct observation of the refracted rays through water holding crystals of eocine or other fluorescent bodies in suspension.

§ 18.   All of these experiments and models show the form of focal interval given in Fig. 13.   Here $V\,V'$ represents the vertical and $H\,H'$ the horizontal meridian.   The rays refracted by

Fig. 13.
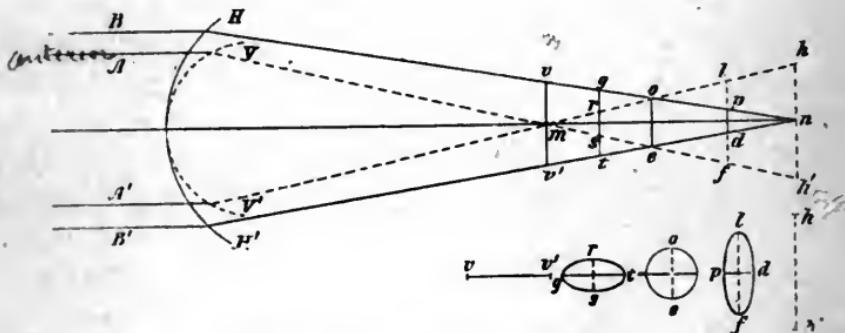
Showing the manner of formation of the Focal Interval of Sturm.

$V\,V'$ are brought to a focus at m, and those rays falling in the planes parallel to it are focussed on the horizontal line $v\,v'$, forming the *anterior focal line* of the interval. These rays, after meeting, diverge in the direction of $h\,h'$. The rays refracted by the horizontal meridian $H\,H'$ pass the line $v\,v'$ before meeting, forming its limits, and finally are brought together at n on the vertical line $h\,h'$. All the rays falling in planes parallel to $H\,H'$ likewise find their foci on this line, which is the *posterior focal line* of the interval, and its limits are the continuation of the rays that have been refracted by $V\,V'$ and have crossed at $v\,v'$.

If this interval be divided by sections perpendicular to the

axis X X' we shall have the forms represented at the lower part of Fig. 13. The figures, it will be seen, are the same as those obtained in the experiment with the combination of spherical and cylindrical lenses shown in Fig. 12.

We first have at the focus of the most strongly refracting meridian v v', the anterior focal line, which is *horizontal* and at right angles to its refracting surface. This gradually merges into an ellipse g r t s, whose long axis, g t, is *horizontal*; this then gradually passes into a *circular* figure o e; then passes into an ellipse p l d f with its long axis *vertical*; and, finally, by a gradual decrease in its short and increase in its long axis, it merges into the posterior focal line h h', which is *vertical*, being at right angles to its corresponding refracting meridian, H H'.

§ 19. It will be discovered on a study of these figures: first, that the anterior is shorter than the posterior focal line. This is due to the fact that in the two similar triangles v v' n and h h' m with a common altitude n m, the angle n opposite to v v' is smaller than the angle m opposite to h h'. This must always be the case, since the angle h m h', being the opposite internal angle of y m y' is larger than v n v'. It follows, moreover, for the same reason, that the difference in the size of these angles, with a consequent difference in the length of their opposing sides, must increase with the lengthening of the focal interval. Second, the circular section of the figure will not be found as is represented in all the figures of the interval I have seen, except those given by Mauthner, and as it is actually described as being by several, in the *middle* of the focal interval, but always nearer the anterior focal plane, for the reason that v v' is under all circumstances shorter than h h'. Third, any ellipse anterior to the circle, on the same account, is shorter than any posterior ellipse taken at the same distance from the posterior focal line as the anterior is from the anterior focal line.

It follows also, from the same reasoning, that the *circles of diffusion are greater on the posterior than on the anterior focal plane.*

§ 20. It is abundantly apparent from these demonstrations that

it is impossible to have, by any such refracting surface, a clearly
defined image of all parts of an object on any one focal plane.   It
is also evident that the degree of astigmatism, or the amount of
deviation from a spherical refraction, is measured by the length
of the focal interval, that is, by the difference in the focus of the
meridians of greatest and least refraction.

Fig. 14.
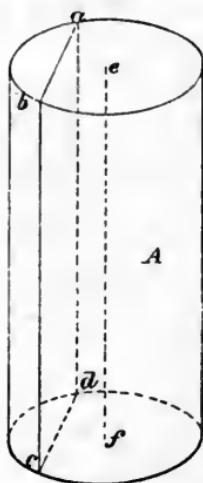
FORMATION OF A CYLINDRICAL LENS.

§ 21.   From the foregoing it is seen that ordinary spherical
lenses would have no effect in correcting the astigmatic condi-
tion.   To remedy the defect in form we need lenses which are
curved only in one direction.   Such lenses are found in *sections
of cylinders made in the direction of their principal axes.*

In the cylinder *A*, Fig. 14, *e f* is the axis, and *a b c d* a sec-
tion made parallel to it.   In this section one surface is plane,
and the other curved, but *the curvature is only in one direction
and that at right angles to the axis.*

The action of such a lens on an incident bundle of rays is
shown in Fig. 15.   The axis of the cylinder is *f*, and the paral-
lel rays *a a'*, *c c'*, *e e'*, etc., falling in planes perpendicular to
the axis will be united, after refraction, in the same planes at

the points a'', c'', e'', etc., on the line $F\,F'$, parallel to the axis of the cylinder f. Those parallel rays falling in planes parallel to f will remain parallel after their passage through the lens.

Fig. 15.
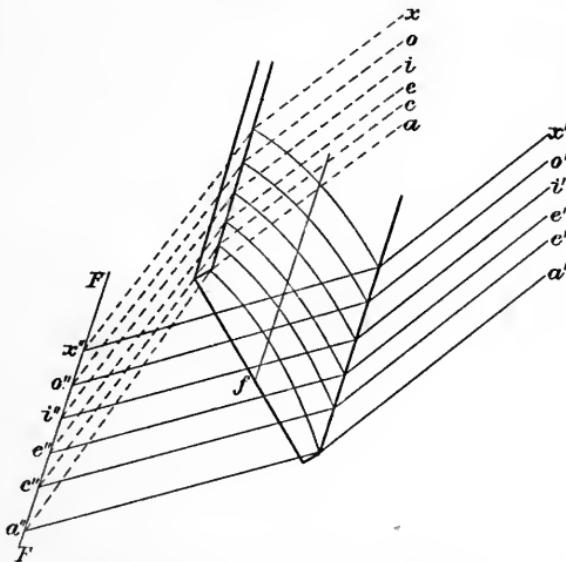
REFRACTION BY A CYLINDRICAL LENS.

Fig. 16.
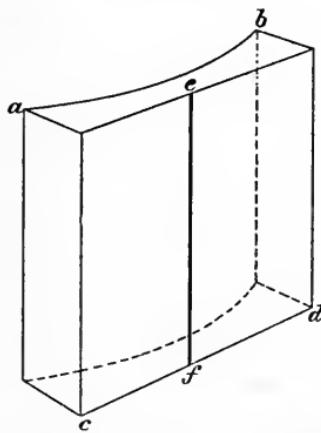
A CONCAVE CYLINDER—$c\,f$ its axis.

§ 22. The curved surface of the cylinder may be either convex, as in Fig. 14, or concave, as in Fig. 16; and as the radius of curvature may be of any length, cylindrical lenses may be

made of any refracting power. They are numbered according to the same system as spherical lenses, (§ 9).

§ 23. As in the case of lenses with spherical surfaces there may be several *forms* of cylinders. When one surface is plane it is called a *plano-convex* or *plano-concave cylinder*; when both surfaces are curved after the same manner it is called *double convex* or *double concave*. These latter forms, however, are seldom used. When one surface is cylindrical and the other spherical the combination is called *sphero-cylindrical*, and when both surfaces are cylindrical, with their axes at right angles to each other, they are called *crossed cylinders*.

When two plano-convex or concave cylinders having the same radius of curvature are placed with their axes at right angles to each other the refracting effect will be that of a plano convex or plano concave spherical lens, its focal distance being that of the cylinders separately. Thus two +2 cys. with their axes at right angles would make a +2 spherical.

When the opposite side of the cylinder is curved it is not necessarily after the manner of its own curvature. Thus the opposing surface of a plus cylinder may have a concave spherical curvature or a concave cylindrical curvature with its axis at right angles to the axis of the positive lens.

§ 24. There is frequently a practical advantage to be derived from certain "over correcting" combinations, and it is possible by this means to do away entirely with "crossed cylinders," which are more difficult of manufacture than sphero-cylinders.

Let us suppose, for example, that we wish to use a +2 D cy. with a —4 D cy., with their axes at right angles. Instead of having one surface ground cylindrically convex, of the required strength, and the other concavely so, with their axes at right angles, we can have the following combination: —4 spherical on one side and +6 cylindrical on the other side. This union would leave the —4 undisturbed in the meridian parallel to the axis of the cylinder, while in the meridian at right angles to it the +6 would neutralize the minus refraction of the 4 D spherical lens and still leave a positive cylindrical refraction of 2 D.

§ 25. When two plane cylinders of opposite refraction are

placed with their plane surfaces together, and revolved about their common center the result of the combination is a constant variation in refracting power, and in the direction of the axis of the astigmatic system.   On this principal Stokes constructed in 1849, the lens which is still known by his name, though there have been modifications made in its mechanism by Javal, Snellen, Dennett and others.

If a plane —3 cy. and a plane +3 cy. are so placed that their axes correspond, the one lens will neutralize the other.   When, however, they are turned about their centers so that their axes are at angles, the astigmatic action will increase and the axis of the combined system will change until the axes of the two lenses are perpendicular to each other, when the total astigmation will be equal to the sum of the power of the two lenses, that is, 6 D.   It was hoped at one time that this apparatus, on account of its compactness and the amount of astigmatic action it was capable of representing, might come into general use in practical ophthalmology.   But the combination gives not only a cylindrical action, but also what amounts to a spherical refraction, which constantly varies with the rotation, and must always be taken into account when examinations are made, as they usually are, with parallel rays.

Dennett, of New York, has recently made a very ingenious modification of the Stokes lens by which it is possible not only to vary the strength of the cylinders, the axes remaining the same, but by turning the same milled head on another set of cogs to vary the direction of the axes.   Both the strength of the cylinders and the direction of their axes are read off on the apparatus.   The whole instrument is conveniently mounted so that it can be manipulated by the patient, and for intelligent persons is most useful for self experimentation.   It is, of course, open to the objection inherent in all crossed cylinders of the description mentioned above. For examination at short range, however, it may prove valuable in speedily obtaining the direction of the principal meridians, and the character and, approximately, the degree of astigmatism.

But a cylindric or astigmatic action is also obtained when a spherical lens is placed obliquely to the path of the rays of light. In fact, the astigmatism of the eye was first referred wholly to the oblique position of the crystalline lens by Young, who was the discoverer of ocular astigmatism. This cylindrical refraction increases with the degree of inclination of the lens.

The amount of this cylindrical action has been computed by Pickering and Williams, and we give in Table II two of their tables. One of these represents the shortening of the focus of a lens of 100 inches focal distance for every five degrees of inclination on the horizontal axis. The other shows the same when the lens is rotated on its vertical axis. These authors explain the discrepancy in the two tables by the fact that in the vertical inclination the rays are no longer in the same plane.

TABLE II.

Horizontal Inclination.		Vertical Inclination.	
i.	*f.*	*i.*	*f.*
0°	100	0°	100
5°	99.9	5°	98.9
10°	99.2	10°	96.1
15°	97.7	15°	91.2
20°	96.1	20°	84.8
25°	93.7	25°	77.0
30°	91.1	30°	68.4
35°	88.3	35°	59.2
40°	84.7	40°	49.8
45°	81.1	45°	40.6
50°	77.2	50°	32.0
55°	73.2	55°	24.0
60°	69.0	60°	17.1
65°	64.7	65°	11.3
70°	60.4	70°	7.0
75°	56.3	75°	3.8
80°	52.1	80°	1.6
85°	48.3	85°	0.4
90°	44.8	90°	0.0

§ 26. The rotation of a cylinder on its axis influences its refracting power. Dr. G. Hay has called attention to this[1] and demonstrated mathematically that the focus of a cylindrical lens is *shortened*

[1] Trans. Amer. Oph. Soc., 1875.

by such a rotation about its axis.   Dr. Sous[1] dissents from this view and gives a mathematical demonstration to the opposite effect, and contends that the lens in rotating about its optical axis is not increased, but, on the contrary, diminished in its refraction.   It is not necessary to enter into a mathematical com-

TABLE III.

Obliquity Observed in Degrees.	Neutralizes Glass.	Obliquity Observed in Degrees.	Neutralizer Glass.
21 32¹/₂ }	$-1/60$	52 53+ 54 }	$-1/18$
27 30 45 }	$-1/48$	54 55 57 }	$-1/16$
37¹/₂ 39 }	$-1/42$	56 58 }	$-1/15$
42 43 }	$-1/36$	56 58 59 }	$-1/14$
47 48 49 }	$-1/30$		
48 59 }	$-1/24$	58 59 61 }	$-1/13$
50+ 50— 52¹/₂ }	$-1/20$	60 61 }	$-1/12$

putation to demonstrate the fact that a cylinder does increase in power by being turned about its axis.   The following simple experiment is sufficient to establish it. If a $+4$ cylinder is placed before the eye, with its axis horizontal, and a series of radiating lines (Snellen's fan) is looked at, only the vertical line will be seen distinctly, the horizontal lines being scarcely visible.   If a -3 cylinder is now placed before this lens with its axis coinciding with that of the plus cylinder, the horizontal lines will become more distinct, but remain much less distinct than the

[1]Traite d' optique, Paris, 1881, p. 464, et seq.

vertical.   There still remains a cylindrical action of $+1$ D un-
corrected.   If the -3 is now rotated on its axis, the horizontal
lines will increase in clearness, and when it has an inclination
of nearly 45° they will be as clear and distinct as the vertical,
and the whole fan will be even, showing that the one lens has
wholly neutralized the other; in other words, a -3 rotated 45°
on its axis has the refracting power of a -4.   We copy Table
III from Dr. Hays' paper showing the negative glass which
a $-^1/_{10}$ neutralizes when placed at various inclinations.

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CHAPTER III.

ASTIGMATISM IN THE HUMAN EYE—HISTORY OF CORNEAL
ASTIGMATISM.—THE DIFFERENT FORMS OF AME-
TROPIA—VARIETIES OF ASTIGMATISM.

§ 27. The human eye, with rare exceptions, suffers, among other imperfections, from astigmatism. Even eyes that have an acuteness of vision which is normal according to a conventional standard, will be found, on careful examination, to be astigmatic in a greater or less degree. When this astigmatism is not sufficient to lower perceptibly the visual acuteness, it is considered *normal*. When, however, it is of such a degree that vision no longer reaches the generally accepted standard of normality, it is regarded as *abnormal*.

§ 28. The refracting media of the eye are the cornea and crystalline lens, and astigmatism may be found in either or both. In addition, therefore, to the division into regular and irregular, astigmatism (§ 12) may be divided into *corneal* and *lenticular*.

Corneal astigmatism is, for the most part, regular; while lenticular astigmatism is, most generally, irregular. To this general rule, however, there are exceptions, which will be considered when each form is treated of in detail.

§ 29. The cornea in normal eyes suffers, in addition to the regular form of astigmatism, in which the opposing meridians have different foci, from a monochromatic aberration in these meridians themselves, which undoubtedly exercises an influence on the distinctness of the retinal image.

The outline of the corneal surface in the optically normal eye has never been determined with exactness, nor the peculiarities of its refraction treated of in a thorough manner. I therefore asked my friend Prof Wm. Harkness, of the United

States Naval observatory, to take the data derived from the measurements of a number of emmetropic eyes, and determine what the actual refraction of the normal cornea is in one meridian of its curvature.

This he kindly did, and the results are published in the *Archives of Ophthalmology*, vol. xii. pp. 9--19. A résumé of his investigations is herewith given:

"The cornea of the emmetropic eye seems to have an ellipsoidal form, but the existing data for determining its curvature in the vertical meridian are too meagre to give a satisfactory result. I have therefore confined my attention to the horizontal meridian, and have taken the data for it from table VII, upon pages 598--599 of Mauthner[1]. That table exhibits the form and dimensions of the cornea in seventeen pairs of emmetropic eyes, and from the mean of these thirty-four eyes, it appears that in the visual axis the radius of curvature is 7.708 millimetres, while 20° to the inner side of that axis it is 8.378 millimetres, and 20° to the outer side 7.884 millimetres. Mauthner does not explain the phrase "20° to the inner (or outer) side of the visual axis," and I have had some trouble in ascertaining its true meaning.

"It is customary to regard the outline of the cornea, along a horizontal section through the visual axis, as part of an ellipse. Let it be part of the ellipse NAMB, Fig. 17, of which the major and minor semi-axes are respectively AH and BH; and let C, D, E, be the points at which the radii of curvature are measured. *DF*, *CF* and *EF* are normals to the ellipse at these points, and *CF* is the visual axis. By the phrase "20° to the inner side of the axis" Mauthner means that the angle *CFD*, between the visual axis and the normal at *D*, is 20°; and, similarly, by "20° to the outer side of the visual axis" he means that the angle *CFE* is 20°. It may be well to remark that Fig. 17 is accurately drawn to scale, and the arc *NAM* represents that portion of the ellipse which is included within the limits of the cornea. * * * * * * * The monochromatic aberration of the

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[1]Vorlesungen ü. d. optis., Fehler des Auges, 1876.

cornea is most conveniently investigated by tracing the paths
which a considerable number of parallel rays, impinging upon
it will pursue within the eye.   Eleven such rays have been con-
sidered, all situated in the same horizontal plane, at intervals

Fig. 17.
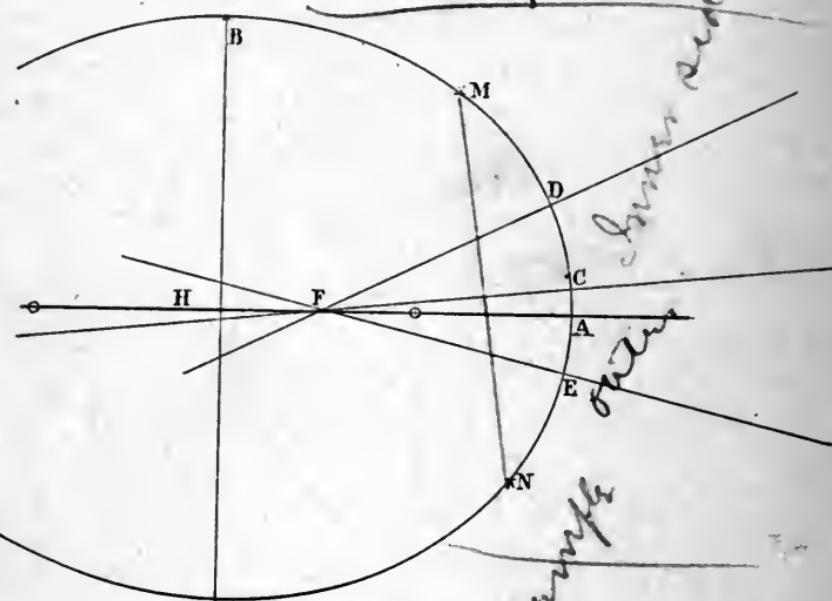
SHOWING THE ELLIPTICAL FORM OF THE CORNEA—$C\ F$ is the Visual Axis;
$A\ H$ the Optical Axis.

of half a millimetre from each other, and the central one coin-
ciding with the visual axis.   For their passage a pupil of five
millimetres in diameter is necessary.   The eleven rays in
question furnish eleven values of d, varying by intervals of half
a millimetre from $+2.5^{mm}$ (towards the nose), to -2.5^{mm} to-
wards the temple), from which the corresponding values of F
have been computed."

The results of these computations are represented graphic-
ally in Fig. 18.   The center line o is the visual axis, while -3,
2, 1, represent the distance toward the temple in millimetres,
and $+3$, 2, 1, the distance towards the nose in millimetres.
The ordinates represent the paths of the eleven rays whose foci
have been computed.   The values of F are taken as abscissas,

and the curved line shows the position of these foci at intervals of one-tenth of a millimetre. From this it is evident that the focal curve is a parabola.

Fig. 18.
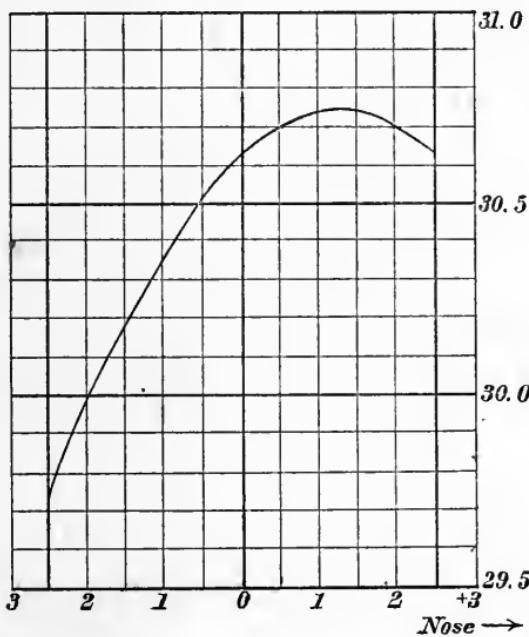
SHOWING THE FORM OF THE FOCAL CURVE OF THE NORMAL CORNEA.

"Table IV shows how the monochromatic aberration actually existing in the normal cornea compares with what would have existed if the cornea had been spherical. Upon each line of the table, the first column, d, contains the assumed diameter of the pupil; the second, third and fourth columns relate to the normal cornea, and contain respectively the greatest and least focal distance occurring within the area of the pupil, and the amount of astigmatism corresponding to them; the fifth, sixth, and seventh columns relate to a spherical cornea, and contain similar data for it. In computing the focal distance at various points of a spherical cornea it has been assumed that the radius of curvature $r = 7.708^{mm}$, and the index of refraction $\eta = 1.3366$.

As the monochromatic aberration we are considering occurs

TABLE IV.

d.	Normal Cornea.			Spherical Cornea.		
	F.	F'.	As.	F.	F'	As.
mm.	mm.	mm.	Par. in.	mm.	mm.	Par. in.
1	30.684	30.502	1:139	30.608	30.556	1:476
2	.723	.359	1: 69	.608	.414	1:130
3	.728	30.182	1: 46	.608	30.171	1: 58
4	.728	29.971	1: 33	.608	29.826	1: 32
5	30.728	29.724	1: 25	30.608	29.376	1: 21

in a single meridian, its correction by cylindrical lenses is impracticable, but nevertheless its amount may be expressed in the notation usually employed for astigmatism. The requisite formula is

$$\text{Astigmatism} = 0.0396 \, (F-F')$$

In which F and F' are the lengths in millimetres of the greatest and least focal distances found within the area of the pupil, and the result is expressed in terms of the Paris inch.

In conclusion, the results at which we have arrived may be summed up as follows:

a. The monochromatic aberration originated by the normal cornea occurs principally on the outer side of the visual axis— that is, on the side farthest from the nose.

b. The diameter of the pupil is usually about four millimetres. For diameters less than this, the monochromatic aberration of a spherical cornea would be less than that of the normal cornea. For greater diameters the reverse is true. This is contrary to the generally received opinion. (See Donders, foot-note on page 310.)

c. Donders says (pp. 456, 457, Anomalies of Ref. and Accom. of the Eye), the astigmatism in sharp eyes is not generally more than from 1:140 to 1:60, and whenever it exceeds the latter amount the power of vision suffers under some circumstances. An astigmatism of 1:40 he regards as decidedly abnormal. Nevertheless, with a pupil four millimetres in diameter the normal cornea produces monochromatic aberration to the extent of 1:33."

Aubert (Archiv. f. d. gesammt Phys. B. xxxv) has recently made some measurements in order to determine more accurately the form of the corneal surface, and has found that at about 12° towards either side of the visual axis it becomes very rapidly flattened, thus dividing the surface into two true zones, the polar and peripheral. The central or polar zone, which is used exclusively for optical purposes, he calls the *optical* zone; the peripheral, bordering on the sclera, he calls the *scleral* zone. The regular curve of the optical zone reaches about 17° to either side of the apex of the cornea, but even in a very wide pupil the rays refracted through the scleral zone cannot enter the eye. In these measurements he has not determined the question of whether the optical zone is spherical or elliptical.

§ 30. But aside from this monochromatic aberration of the cornea in a single meridian (which must be considered as forming a part of its irregular astigmatism) there exists, in most eyes, a difference in the curvature of the principal meridians taken as a whole, constituting *regular astigmatism.*

A number of eyes with normal vision have been measured as to the curvature of their various corneal meridians, and in almost all there has been found a meridian of least curvature (longest radius), and at right angles to this another of greatest curvature (shortest radius). In other words, the human cornea does not represent, by its surface, the section of a sphere with equal radii, but approaches in form more nearly to an ellipsoid with three unequal axes, such as we have studied in Chapter II.

In the table V are given the results of measurements of twenty-one normal eyes, showing the amount of astigmatism from which they suffer.

It will be seen from an examination of this table that in only two eyes were the two principal corneal meridians the same. In the other nineteen there was a difference, estimated by the focal distance of the lens necessary to correct it, and expressing the amount of the astigmatism, of from 1:280 to 1:38.

TABLE V.

No.	Observer.	Radius in Horizontal Meridian.	Radius in Vertical Meridian.	Focus in Horizontal Meridian.	Focus in Vertical Meridian.	Astigmation = 1:
		Mm.	Mm.	Paris Inches.	Paris Inches.	Paris Inches
1	Knapp.	7.80	7.91	1.1445	1.1605	—62
2		8.07	8.26	1.1840	1.2120	—40
3		7.23	7.385	1.0688	1.0835	—38
4		7.22	7.08	1.0593	1.0388	40
5		7 74	7.71	1.1356	1.1313	220
6		7.74	7.74	1.1356	1.1356	
7		8.20	8.12	1.2031	1.1914	88
8		8.34	8.19	1.2237	1.2107	85
9	Donders and Doyer.	7.23	7.23	1.0608	1.0608	
10		8.27	8.30	1.2134	1.2178	—250
11		7.73	7 69	1.1342	1.1283	160
12		8.15	7.94	1.1958	1.1650	34
13		8.08	7 81	1.1855	1.1457	29
14		8.02	7.92	1.1767	1.1626	76
15		7.42	7.30	1.0887	1.0711	50
16		7.49	7.51	1.0987	1.1019	—280
17		7.49	7.45	1.0987	1.0931	160
18		7.84	7.46	1.1503	1.0946	16.9
19		7.75	7.33	1.1371	1.0755	14.9
20		7.60	7.53	1.1151	1.1048	89
21		7.55	7.60	1.1078	1.1151	—127

It should be stated in this connection, however, that the corneal astigmatism does not in all cases represent the exact astigmatism from which the eye suffers. There may be a concomitant lenticular astigmatism which will increase or diminish that of the cornea according to the direction of its faulty meridian and the degree of its astigmatic deviation. The total astigmatism of the eye is the algebraic sum of its corneal and lenticular astigmatism. This we shall deal with further on under the head of lenticular astigmatism.

It will be furthermore observed that in thirteen cases the vertical was the *more strongly curved meridian*, the horizontal being the stronger in only six cases (those marked — in the table). Clinical observation being in accord with this, it is customary to speak of this relation of the principal meridians in which the vertical is the more strongly curved as being *according to the rule*.

§ 31. The existence of a slight degree of astigmatism in a normal

eye can be experimentally demonstrated by turning before it a weak cylindrical glass—say, with a focal distance, positive or negative, of 144 inches (0.25 D). In this experiment it will be found that when the axis of the cylinder corresponds to one certain meridian vision will be best, and when it corresponds to the one at right angles to it it is worst. In the first instance the cylinder corrects the existing astigmatism, and, perhaps slightly *over* corrects it, while in the latter it increases it by the power of the lens.

If two very fine threads are crossed at right angles and brought gradually towards the nearest point of distinct vision, it will be found, usually, that one becomes blurred before the other. The point where the first thread becomes indistinct marks the near point of the weakest-refracting meridian, the point where the line at right angles to it becomes blurred is the near-point of the most strongly refracting meridian. The difference between the two points measures the amount of astigmatism. This is the method used by Young in demonstrating his astigmatism.

§ 32. Regular astigmatism *may also have its seat in the lens*. Thomas Young, who was the first to demonstrate the existence of astigmatism in the human eye, found his own astigmatism to reside there. He immersed his eye in a chamber of water bounded by a plane glass surface, thus eliminating the refraction of the cornea, and, as the difference in the refraction of the two meridians did not disappear, he rightly concluded that it must be due to an abnormality of the lens.

The *causes* of lenticular astigmatism are either displacement of the lens in such a manner that its refracting surfaces shall lie obliquely to the visual axis (§ 25), or a difference in curvature of its principal meridians, as in corneal astigmatism. Young conceived his astigmatism to be due to an obliquity of the lens.

A simple displacement of the lens at right angles to the visual axis would not of itself produce astigmatism.

There have been some cases reported of progressive change in the degree of astigmatism and in the direction of the prin-

cipal meridians. In none has there been any measurement of
the corneal radius, and under these circumstances we cannot
positively exclude a change in corneal curvature, but it is most
probable that the change is due to alteration in the shape of
the lens, either from changes in its substance or from a modi-
fied action of the ciliary muscle.

§ 33. It is probable that Gerson (1810) was the first to point out
the existence of corneal astigmatism, but I do not find that the
opinion was based on ophthalmometric measurements, and his
statement only became generally known after the fact had been
established by the measurements of others. Wilde and Jones
refer to "cylindrical eyes" and "cylindrical cornea," but there are
no evidences that these opinions were based on opthalmometric
measurements.

Wilde says[1]: "It is well known that the cornea is not a cor-
rect surface of revolution, but that the curvature of its hori-
zontal plane is less than that of its vertical. When this ex-
ceeds the normal extent it gives rise to irregular refraction,
causing a circle to appear oval."

Jones simply quotes from the history of Airy's case. No
practical application of these facts seems to have been made
by either of these writers.

Senf was the first (in 1846) to make measurements of the
cornea, which showed it to be ellipsoidal rather than spherical
in shape. Helmholtz arrived at the same conclusion from his
ophthalmometrical measurements which were published in
Gräfe's Archives B. i, Abt. 2 (1855). In this article (p. 18) he
says: "The form of the cornea corresponds approximately to
an ellipsoid formed by the revolution of an ellipse about its
major axis."

He gives these measurements as well as those of Senf in the
first part of his " Physiologische Optik " (pages 8 and 11 of
the French edition), published in 1856; but it is evident that
he still considered the cornea to be an ellipsoid of revolution,
as his measurements were confined to one meridian (the hori-

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[1] Dub. Jn'l Med. Sci. 1846-47. P. 105.

zontal). He speaks at this time (p. 142) of the astigmatism of Young as being caused by the lens, and of the correction of his own astigmatism (of low degree) by means of an obliquely placed concave lens, but no hint is given that the cause of the astigmation was in the cornea.

It was Knapp who first determined by ophthalmometric means that the cornea was not an ellipsoid of revolution but an ellipsoid with three unequal axes.

In the "Verhandlungen der vom 3-6 Sept.,1859, im Heidelberg versammelten Augen Ærtze," Berlin, Peters, 1860, we find (p. 19) that "Dr. Knapp gave an account of his measurements on the curved surface of the human eye, made by means of Helmholtz's ophthalmometer. 1. The cornea. Helmholtz's measurements were confined to the horizontal meridian. Knapp, on the other hand, had measured four eyes in many different meridians with the following result: 1. The centre and apex of the cornea do not coincide. * * * The anterior focal distance of the horizontal ellipse =23.095 *mm.*; of the vertical ellipse =23.34 *mm.* The posterior focal distance of the horizontal ellipse =30.18 *mm.*; of the vertical ellipse =31.1 *mm.* * * * In the discussion on this division of the subject, Knapp remarked that in all probability it was the difference between the vertical and horizontal ellipses which rendered cylindrical glasses necessary, and was the cause of the difference in the 'accommodation line' in the vertical and horizontal directions. After cataract-extraction, in sclerectasia and hyperpresbyopia, such glasses were of benefit, as had been shown by Prof. Donders.

"In regard to the accommodation-line, Prof. Donders remarked that, in his opinion, it was due to the lens, from the fact that it was in intimate connection with polyopia, which was undoubtedly caused by the lens, as proven by entopic experiments."

These investigations were published in detail by Knapp in his inaugural thesis, "Die Krümmung der Hornhaut des menschlichen Auges" in 1860.

Donders, in his first papers on the refraction and accommo-

dation of the eye, published in *Gräfe's Archives*, makes in
B. vii, Abt. 1, p. 176 (1860), an application of this asymmetry
to the explanation of abnormal astigmatism, in contradis-
tinction to the lenticular theory of Young, and gives reference
to Knapp's paper.   So far as I know this is the first mention
made by Donders of corneal astigmatism.   In *Gräfe's Archives*
B. viii, Abt. 2 (1862), appeared Knapp's classical paper, "Ueber
die Assymmetrie des Auges in seinen verscheidenen Meridi-
anebenen."   While this paper was in press Donders published
his "Astigmatisme en cylindrische Glazen," which, for the
first time, brought the subject of astigmatism and its cor-
rection prominently before the profession.   Soon afterward
(1864), his treatise on the "Anomalies of the Refraction and
Accommodation of the Eye" appeared, which made astigma-
tism a part of the general knowledge of the profession.

The opinion that regular astigmatism resides almost wholly
in the cornea has been most thoroughly substantiated by all
observations made since that time.   Javal in the *Annales d'*
oculistique, t. 87, pp. 33-43 (1882), says that in the testing and
measurement of more than 100 eyes, the total astigmatism
corresponded exactly with the corneal astigmatism, with the
exception of four cases, and in one of these the difference was
only 0.2 D; and additional examinations by him, and by Dr.
Nordenson, amounting to more than 250 cases, have confirmed
his first observations.

My own experience with Javal's ophthalmometer tends to
substantiate this opinion in the main, though my percentage of
difference between the total and corneal astigmatism is greater
than that given by Javal and Nordenson.

§ 34.   As stated in § 27, when the astigmatism reaches such
a degree as to reduce the visual acuteness below the normal
standard of $\frac{4}{4}$ ($\frac{20}{20}$) it is called *abnormal*.

Exactly what degree of astigmatism should be considered
abnormal has not been agreed upon by observers.   Some look
upon $\frac{1}{160}$ (0.25 D) as abnormal, while others think $\frac{1}{72}$ (0.50)
normal.   It is impossible to formulate any law of universal ap-
plication for such a varying and imperfect optical instrument

as the eye.   Prof. Harkness has shown us (§ 29) that the normal irregular astigmatism of a single meridian does not probably fall below $1/_{33}$, and since a large majority of persons cannot distinguish any sensible difference in the distinctness of the image of test objects as fine as the finest test-types when a correcting 0.25 is placed before their eye with its curvature corresponding to the faulty meridian, we *feel justified in considering only those degrees of astigmatism abnormal which exceed 0.25* ($1/_{160}$). We would, however, not deny the possibility of certain rare cases deriving benefit from the employment of glasses of this low power.

§ 35.   When we come to study astigmatism as it affects the human eye in detail, we find that we have several different conditions to deal with, and in order to clearly understand the manner and degree of departure of the astigmatic from the optically normal eye we must know in what the latter consists.

§ 36.   A normal, standard or *emmetropic* eye (that is, an eye of proper measure) is one whose retina lies at the focus of its refracting media.

Fig. 19.
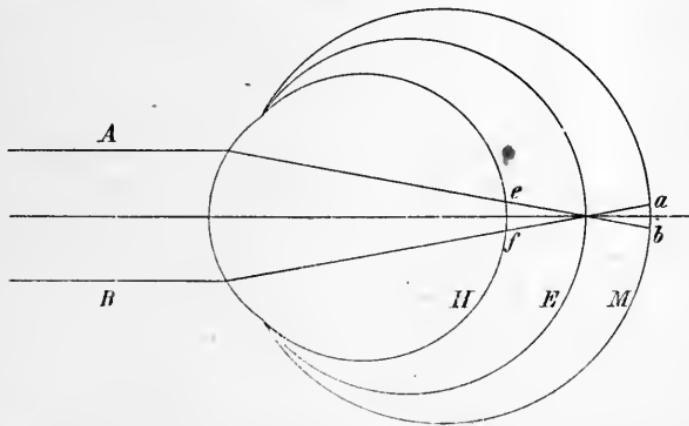
THE EMMETROPIC AND AMETROPIC EYES COMPARED—*H*, the hypermetropic ; *E*, the emmetropic ; *M*, the myopic eye.

Parallel rays falling on all meridians of such an eye are brought to a focus at the same point on the retina, *E*, Fig. 19.

§ 37.   An eye which deviates from these standard refractive conditions is called *ametropic* (not of proper measure).

When the parallel rays are focussed *before* the retina, *M*, Fig. 19, the eye is called *myopic*. When they cross *behind* the retina situated at *H*, the eye is said to be *hypermetropic*.

§ 38. In the ordinary myopic and hypermetropic forms of ametropia it has been found from ophthalmometric measurements, that, as a rule, the radius of curvature of the cornea is the same for them as for the emmetropic eye. These ametropic conditions are due, except in rare cases, to a displacement of the retina; in other words, to a variation in the length of the eyeball. *A myopic eye is one that is too long; a hypermetropic eye is one that is too short.* Ordinary myopia and hypermetropia are, therefore, *not strictly speaking anomalies of refraction*, since the refracting power of these eyes is normal as compared with that of the emmetropic eye. The refraction is abnormal only when considered in reference to the position of the retina.

In those exceptional instances where the refraction is at fault its seat may be in the cornea or the lens.

§ 39. Astigmatism is the only anomaly which is due entirely to a defect in the refracting apparatus, for, as we have seen, it is the faulty curvature of the refracting media which is the cause of the optical error.

§ 40. We can now understand how astigmatism, in addition to its own error in refraction, may be complicated with the other forms of ametropia.

The foci of both meridians may lie in front of the retina, as in myopia, or they may lie behind it, as in hypermetropia, or one may be in front and the other behind it, constituting myopia and hypermetropia at once.

It is, therefore, important not only to facilitate study, but, as we shall see, for the practical purpose of correcting the anomaly, that we make subdivisions of astigmatism. The general forms into which it has been divided are: 1. The simple. 2. The compound. 3. The mixed.

§ 41. SIMPLE ASTIGMATISM. When the focus of one meridian falls on the retina the astigmatism is called simple. There can, of course, be but two varieties of this form, one in which

the focus of the faulty meridian falls *in front* of the retina, and the other in which it falls *behind* it. Borrowing the nomenclature of spherical ametropia, the first condition is called *simple myopic* astigmatism, and the second, *simple hypermetropic astigmatism*. One meridian is emmetropic, the other myopic or hypermetropic.

§ 42. COMPOUND ASTIGMATISM. In the compound form both foci fall either in front of or behind the retina. When the foci lie *before* the retina it is called *compound myopic astigmatism;* when they both lie *behind* it there is *compound hypermetropic astigmatism*. In this form, therefore, we have an astigmatism associated with spherical ametropia. Both meridians are ametropic, and in the same manner, but one of greater degree than the other.

§ 43. MIXED ASTIGMATISM. This is the condition where the focus of one meridian lies *in front* of and the other *behind* the retina. In other words, one meridian is *myopic* and the other *hypermetropic*. Of this form there can of course be but one variety.

Every case of regular astigmatism must be of one of these three forms.

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CHAPTER IV.

———

Diagnosis of Astigmatism—Determination of its Form
and Degree, and the Direction of the
Principal Meridians.

§ 44. When a case of supposed astigmatism presents itself
for examination there are four points to be settled: *a*, whether
any astigmatism exists; *b*, if it does, the direction of the prin-
cipal meridians; *c*, the special form of the anomaly, and *d*, its
degree.

§ 45. For simplicity of illustration we will assume that in
the case under consideration there does not exist any turbidity
of the refracting media, any affection of the nervous appara-
tus, or any of the complications which we shall consider in
detail later on, but will regard it as a purely optical difficulty.

§ 46. In order to determine the first point, we place the pa-
tient, as we always do when examining the static refraction of
an eye, at a distance of 15 or 20 feet (4 to 6 metres) from the
well-known test types of Snellen. The acuteness of vision of
each eye is then taken separately; and if we find that all the
letters in No. xx (6), are clearly made out at a distance of 20
feet, the existence of astigmatism in any abnormal degree
can be excluded.

§ 47. If, however, V does not reach $^6/_6$ we may be sure of
the existence of some error in refraction, which may be either
myopia, hypermetropia or astigmatism.

In order to determine which kind of ametropia is present,
we place in front of the eye first, a+1 spherical lens. If this
does not render the vision worse, or if it, on the contrary, im-
proves it, we are sure hypermetropia exists, and the *strongest* +

lens, through which No. 6 is seen clearly and distinctly, marks the degree of the hypermetropia, and, as $V=^6/_6$, we may be sure that astigmatism does not exist.

. If + lenses do not improve, but, on the contrary, impair vision, then we try spherical concave glasses, and if we find one which gives $V=^{20}/_{20}$ we may safely conclude that it is a case of simple myopia, and the *weakest* concave glass, which gives a normal acuteness of vision, marks its degree.

§ 48. But if spherical lenses, while improving V somewhat, do not bring the acuteness of vision up to the normal standard of $^{20}/_{20}$, we may rightly consider that astigmatism, either regular or irregular, is present.

§ 49. Persons affected with regular astigmatism usually give us an intimation of their peculiar refractive error by the mistakes they make in naming certain letters of the test-types— P and F, for example, are very likely to be confounded; C is often called G, and both are sometimes mistaken for O, while such letters as L and T are readily recognized.

Dr. W. S. Little has devised a "test-card[1] of words, made up of letters confusing to the astigmatic eye," on this principle, which is useful for the purpose of indicating to us whether or not astigmatism exists.

§ 50. Being satisfied that astigmatism is present, the next step is to determine the direction of the principal meridians. One of the simplest methods of ascertaining this is to turn a cylindrical glass of + or — 1 D before the eye, noting the meridians with which the axis of the cylinder corresponds when the smallest test letters of Snellen that can be distinguished are most and least distinct. These meridians, which will be found at right angles to each other, are the meridians of greatest and least refraction.

§ 51. Another plan is to place before the patient a series of lines of equal thickness, radiating from a common centre, such as the well-known fan of Snellen. These lines should be placed at such a distance that one or two running in one direction shall be seen clearly and distinctly. The direction of

[1] Published by J. W. Queen & Co., Phila.

these lines will correspond to one of the principal meridians, and the other meridian will be at right angles to it.

There are other methods for determining the direction of the principal meridians, of which we shall speak later, but in simple, uncomplicated cases these will suffice.

§ 52. The directions of the principal meridians are expressed in degrees representing their inclination to the horizon. Thus: If we find that the line of Snellen's fan which is most distinct is the vertical or at 90°, the corresponding meridian will be horizontal or at 180°, and the opposing meridian vertical or at 90°. If the axis of the cylinder which renders the test-types clearest is at 45° we know that it is the meridian at 135° which is affected by its refraction, for *the axis of the cylinder is always at right angles to the meridian whose refraction it affects.* It therefore becomes easy to express exactly in degrees of inclination to the horizon the direction of the meridians of greatest and least refraction.

§ 53. Having in this way obtained the direction of the principal meridians, it remains to determine whether only one or both are ametropic, and the degree of the ametropia; in other words, to define the form and amount of astigmatism.

This will be most easily done by taking some cases representing each of the three forms of astigmatism.

§ 54. CASE I. The patient has V=$^{20}/_1$ barely, and it is not brought up to $^{20}/_{xi}$ by any + or − glass. By rotating a +1 cylinder before the eye, we find that when the axis is perpendicular or at 90° V=$^{20}/_{xxx}$ and a few letters of No. XX are made out. When the axis is at 180°, vision is very much worse, so that he can scarcely make out No. LXX. This shows that the principal meridians are vertical and horizontal, and also that the horizontal meridian which corresponds with the refracting meridian of the correcting cylinder is hypermetropic. We now place a +0.5cy axis vertical before the eye, which, while improving vision somewhat, does not increase it as much as the +1. The hypermetropia of the meridian is therefore greater than 1 D. We then try +1.5 cy, axis 90°, and find that with this all the letters in No. XX are seen distinctly, while with +2 cy axis 90° they are less distinct. Diagnosis: *Simple hypermetropic astigmatism* in the horizontal meridian. Or we may express it thus: H. astig. 1.5 D axis 90°; for it must always be borne in mind that the axis of the cylinder is at right angles to the direction of the meridian it corrects.

§ 55. CASE II. The patient in looking at Snellen's fan placed at a distance of six metres, says he sees only one or two black lines to the right, all the others appearing blurred. You find upon examination that the line which is sharpest and black-

est is at 125°. You now place before the eye a +1 spherical lens and ask him whether this line still remains as distinct as before. If he answers "No," then you try minus spherical lenses in the same way and if the line is not *more* distinct with than without them, this meridian (35°) is *emmetropic*. You then place plus glasses before the eye and ask their effect on the lines to the left which are most blurred, and if you find that they do not help, but on the contrary render them still more indistinct, you try minus spherical glasses. With these you find the indistinct lines to brighten up, and finally with —2 spherical the lines at about 35° become very clear and sharp, while those at 125° are rendered gray and indistinct. This shows that the meridian with its axis at 35° has a myopia = 2 D. If a cylindrical lens — 2 D is placed with its axis at 35° the fan becomes even, all the lines having the same clearness and distinctness and with it V = $^{20}/_{xx}$. This is a case of simple

Fig. 20.
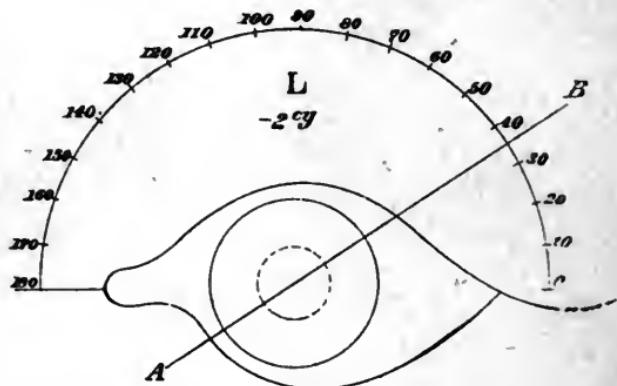
DIAGRAMMATIC REPRESENTATION OF THE DIRECTION OF THE AXIS OF AN ASTIGMATIC MERIDIAN.

myopic astigmatism: M. astig. = 2 D axis 35°, and it may be recorded graphically as in Fig. 20, where the line AB represents the axis of the faulty meridian.

§ 56. CASE III. Without any glass V = $^6/_{36}$, and none of the lines in Snellen's fan are seen with distinctness, all being a confused blur. Spherical glasses are tried as in the other cases, beginning with the convex, and it is found after a number of trials that with a —1.5 all the letters in No. 18 are properly made out, with the exception that P is called F, and with this lens the line in Snellen's fan at 60° is quite sharply defined. Those near the bottom to the right, however, are very indistinct. If this is the *weakest* concave glass through which the line at 60° is sharply defined, then the meridian with its axis at 60° is myopic to the extent of 1.5 dioptries.

It is found on further trial that the line at 150° is brought out clearly by —3, all the others appearing less distinct. This shows that the meridian whose axis lies at 150° is also myopic. Both meridians are therefore myopic, but one in a higher degree than the other. The difference in the degrees (3 — 1.5 = 1.5) of myopia in the two meridians constitutes the astigmatism of the eye. Because there is a spherical ametropia associated with the astigmatism, this form is called *compound myopic astigmatism*, and we record it thus: M. 1.5 D with M. astig. 1.5 axis 150°. With this combination of minus glasses the fan appears uniform and vision = $^6/_6$.

§ 57. CASE IV. We find that the patient, who is a boy of 15, sees the line of the fan at 170° more clearly than any of the others. He can also see it clearly with a $-1/16$ or a $+1/18$ spherical. This shows that there is hypermetropia in the corresponding meridian, and that he overcomes the — glass by means of his accommodation, which is very strong at that age. A $+1/16$ or $+1/15$ blurs this line, therefore meridian with its axis at 170° has H $= 1/18$. With this glass the indistinct lines on the left near the centre also appear brighter and with a $+1/10$ they are perfectly clear, the blackest one being at 80° ; that at 170° is very much blurred. We have, therefore, a H. in the meridian 80° which is greater than that in the meridian at right angles to it. There being hypermetropia in both meridians but more in one than in the other, this is *compound hypermetropic astigmatism*, and as both have H. $= 1/18$

Fig. 21.
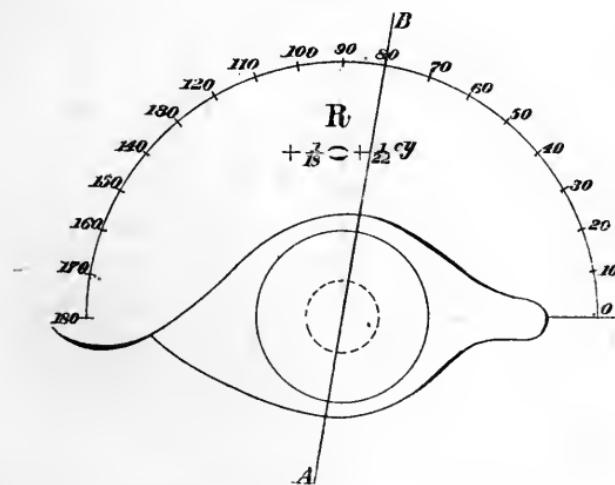
METHOD OF RECORDING A CASE OF COMPOUND ASTIGMATISM.

in common, we write H.$=1/18$ with H. astig.$=1/10-1/18=18/180-10/180=8/180=1/22$, axis 80°, or represent it graphically as in Fig. 21. This example shows the great disadvantage of the old inch system in making subtraction and addition of lenses, which we have so frequently to do in determining astigmatism.

With this combination of lenses V $= 20/20$ and all the lines in Snellen's fan are distinct.

§ 58. CASE V. The patient has very bad vision, not being able to make out No. 60 at 6 metres. Some —glasses, $1/5$, $1/6$ or $1/7$, increase it slightly and $+$ glasses up to $1/22$ do not make vision worse. On asking him to look at the fan we find that all is indistinct, but he thinks he sees a vertical line, though it is much blurred. We place spherical glasses in succession before the eye as usual, and after many trials find that the line at 90° comes out sharply with $+1/22$, while those at the bottom are so confused that they cannot be recognized as lines at all. We are now assured that 90° is the axis of the hypermetropic meridian. The meridian whose axis is at 180° we know is not hypermetropic, because the horizontal lines are more blurred through the convex lenses. We therefore try minus glasses for this meridian, and after a

number of trials find that the line at 180° is sharp and black with —$1/6$, the other lines appearing as a grey blur.   There is therefore a M. of $1/6$ in this meridian. There being H. in the horizontal meridian and M. in the vertical meridian we have to deal here with a case of *mixed astigmatism* and we write H. = $1/22$ 90° with M. = $1/6$ 180°. The *total astigmatism* is therefore equal to the *sum* of these, that is $1/22 + 1/6 = 6/132 + 22/132 = 28/132 = 1/4.75$.   With this combination of lenses (+ $1/22$ 90° ⊃ —$1/6$ 180°) V = $6/6$ and all the lines in the fan become of uniform clearness.

§ 59. In simple, uncomplicated cases the methods employed in the foregoing examples are sufficient, if carefully followed out,to establish the diagnosis and the character of astigmatism, and, under all circumstances, by whatever method the diagnosis of the astigmatism may have been determined, it must be verified by means of cylindrical lenses and the test-types.   The *best vision* is what we aim at, and no method has yet been found which can dispense with this as the final arbitrament.

§ 60. But, unfortunately, all cases of astigmatism are not uncomplicated, and there are many ways in which error, both on the part of the patient and surgeon, may creep in.

The sources of these errors will be considered in the next following chapter.

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CHAPTER V.

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DIFFICULTIES AND OBSTACLES IN THE WAY OF AN ACCURATE
DIAGNOSIS OF ASTIGMATISM—INFLUENCE OF ACCOM-
·MODATION—THE USE OF MYDRIATICS.

§ 61. As stated in the concluding paragraph of the last
chapter, all cases of astigmatism are not so readily determined
as might appear from the examples given. The inexperienced
beginner will meet with many perplexities, and there are cases
which test the skill even of the most expert.

We shall endeavor in this chapter to point out the principal
obstacles that lie in the way of a correct and speedy diagnosis
and the best methods for overcoming them.

§ 62. Most of the difficulties arise from the fact that we are
not dealing with an optical instrument alone, but also with an
·anatomical organ having a physiological function. It is the
office of the eye, as an organ of sense, to interpret impressions
made on the retina, and the judgments formed from these im-
pressions are not always correct. It is astonishing how often
otherwise intelligent persons will make statements as to what
they see, which we know are not true, and which, upon further
questioning and coaching, they reverse. As a matter of ex-
perience, we find that most astigmatic patients have to be, to a
greater or less extent, educated in the right method of observ-
ing and of correctly reporting what they see, and it often re-
quires the exercise of much patience on the part of both ex-
aminer and patient to arrive at the exact truth. The answers
to questions are frequently misleading, and much tact is fre-
quently necessary to extract the true meaning from statements
which, though honestly given, are incorrect. And besides, the
answers are often of such a vague and general character that

opposite interpretations might be given to them, and it is never safe to trust to the results of a first examination. Several trials should be made with different lenses with their axis at various degrees until there is found one which, when placed at the same angle, always gives the same and the best result.

When, in a compound myopic astig., the common M. being 3D, a minus spherical lens of that strength is held before the eye and the patient is told to look at the fan he will most likely say that he sees nothing. Upon close questioning, however, we find that he *does* see a *single line*, and this single line is perhaps the key to the situation. And when, on the other hand, in simple astigmatism of either form, a cylindrical lens of the proper kind, but under or over-correcting, is so placed before the eye that its axis shall correspond with that of the faulty meridian, the patient may exclaim, "Now I see all the lines." That may be true, but it may not be all we want. It is necessary not only that he see them all, but that he see them all with equal clearness and distinctness. Other cylindrical lenses of the same character must be tried until one is found with which the fan is said to appear uniform. But even this should not be entirely trusted. The test-letters must be the final resort, and glasses weaker and stronger than the one selected must be tried to see whether vision is made better or worse by them.

§ 63. We find, under certain circumstances, an improvement with cylindrical glasses when there is no astigmatism present. A person having $M = \frac{1}{15}$ with $V = \frac{20}{c}$, will find vision so much improved by a $-\frac{1}{15}$ cy. that some letters in No. L can be made out. This improvement does not come from the correction of an astigmatism, but is due to a correction of the myopia in one meridian, thus transforming a M. of $\frac{1}{15}$ into a M. astg. of $\frac{1}{15}$, in which one meridian is emmetropic; and an ametropia in a single meridian is much better for vision than an ametropia in both. Under these circumstances, too, vision remains the same when the lens is rotated with its axis in various meridians, which is not the case in astigmatism.

In making examinations by these methods, therefore, *trials*

with spherical glasses should always be made first, and if they do not bring vision up to the normal standard then search should be made for astigmatism.

§ 64. It is believed by some that astigmatics used by preference the centre of their focal interval, or rather that portion whose section is a circle, as shown in E, Fig. 12. This can hardly be, for the eye instinctively seeks to have a distinct image, if it is possible, of some part of an object, rather than a confused image of the whole. It is natural, therefore, that they should use one or the other focal line when it is possible for them to do so. Which focal line they prefer when they have both at command has not been determined definitely, but it would seem that *theoretically* they would select the *anterior focal line*, because here the circles of diffusion are smaller than at the posterior focal line (§ 19). Certain observed cases appear to corroborate this view. I have seen several cases of simple hypermetropic astigmatism with the axis at 90° which had been converted by a tonicity of the ciliary muscles into a simple myopic astigmatism axis 180°, thus rendering 90° emmetropic. I accounted for it by supposing that they preferred to always have vertical lines distinct. It may be stated, in this connection, that the small degree of accommodation possessed by some aphakial eyes which enables them to have an amount of distinct vision with the same glasses at different distances, has been explained by an astigmatism which enables them to see one part of an object more clearly at one distance and another part at another. Their range of A would be expressed by their degree of astigmatism.

§ 65. But aside from these sources of error there is another still more important, which arises from the power possessed by the eye of changing its refractive condition at will. This faculty of "accommodation" (A) resides in the ciliary muscle, and the change is brought about by its action on the crystalline lens, causing it to become more convex, thus increasing its refracting power. It will be seen, on a moment's consideration, that the accommodation becomes an important

factor in determining the static refraction of the eye—that is the refractive condition when in a state of absolute repose. In examinations of the eye as regards its optical properties its dynamic refraction must always be held in mind as a possible element, and its influence allowed for. This is not the proper place to consider the influence of the accommodation on all the forms of ametropia, so we shall limit ourselves to the effects as we find them in astigmatism.

§ 66. Since the effect of the accommodation is to increase the refraction of the eye, we know in what direction to look for its influence. Such an increase of refracting power would diminish the degree of a hypermetropia, convert it into an emmetropia, or even into a myopia. It would change an emmetropia into a myopia, and where myopia existed increase its degree. It would in the same manner change not only the degree, but also the form of an astigmatism. A compound hypermetropic astigmatism can be changed by the act of accommodation into the simple hypermetropic form, into the simple form, or, it may be, into the compound myopic form. Mixed astigmatism may be masked by the accommodation, being converted into the simple or the compound myopic form.

Take for example, H = 3 ⌒ H. astig. = 2 axis 90°. An amount of A equal to 3 D would convert this into a H. astig. = 2 D; that is to say, an increase of refraction of 3 D would render the meridian with its axis at 180° emmetropic and leave a H = 2 D at 90°. If A = 5 D, then the meridian with its axis at 90° would be rendered emmetropic while that at 180° would be myopic by 5 − 3 = 2 D, converting it into a case of simple M. astig. Should A = 6 D it would be a compound myopic astig.: M = 1 ⌒ M. astig = 3 axis 180°.

In testing for astigmatism, therefore, due care must be exercised in eliminating any complications on the part of the accommodation. It is for this reason that we use, by preference, those methods of examination in which the patient is removed to 5 or 6 metres from the test objects. Under these circumstances the emmetropic eye is in a state of repose and adapted to the parallel rays which come from objects at that

distance, and there is no incentive for the exercsie of the accom-
modative power; while in the case of myopia the accommoda-
tion would be of no avail, since the myopic eye has al-
ready an excess of refraction, and its far point is nearer than
20 feet. So we have, in this method, to take only a possible
hypermetropia into consideration. Under ordinary circum-
stances, and where there is no actual spasm or undue to-
nicity of the ciliary muscle, there need be no important error
from this source, if proper care is exercised and sufficient time
is given to the investigation.

§ 67. *Examinations should always begin with convex lenses ;*
and myopic conditions should never be accepted unless there
is an *improvement* with concave glasses, which no other glasses
give. When, for example, we find that — 1 cy. with the axis
at 180° brings vision from $^{20}/_{50}$ to $^{20}/_{20}$, we must not conclude
that we have to do with a case of simple myopic astigmatism,
particularly if the patient be a young person with an active ac-
commodation. It might be that the — 1 cy. 180° had con-
verted a simple H. astig. of 1 D 90° into a simple hypermetro-
pia of 1 D, which the accommodation could readily overcome.
In this instance it is true the astigmatism would have been cor-
rected, but at the expense of the accommodation power. So
before concluding a diagnosis of M. astig. we should try the
effect of a + 1 cy. 90°, and if with this V = $^{20}/_{20}$ we know
positively that it is H. astig. of 1 D axis 90°.

§ 68. There is a way by which we can with certainty get
rid of the errors that arise from the accommodotion. By par-
alyzing the ciliary muscle by some of the mydriatics such as
atropine, duboisine, homatropine, etc., we eliminate its active
or dynamic refraction, and place the eye in a condition of
static refraction, though, as we shall see later, this is not al-
ways its *normal* optical state of repose. When a drop of a 2
or 4% solution of atropine is put in the eye, in from twenty
minutes to half an hour there is great dilatation of the pupil,
followed some minutes later by a loss of the whole or a greater
part of the power of accommodation. When there is spasm
or undue tonicity of the ciliary muscle it frequently requires

several instillations practiced at intervals of an hour or so to produce a complete relaxation of the muscle, and sometimes it takes several days, with from four to six instillations each day, to obtain the full effect of the drug.

§ 69. But while this paralysis of the ciliary muscle gives us a relief from disturbances on the part of the accommodation, it is in itself not entirely free from disadvantages, inconveniences, and even errors.

One of the chief inconveniences attendant upon paralysis of A is that the effect of the mydriatic does not pass off fully within a week, and often ten or twelve days elapse before the ciliary muscle regains its normal tone. During this time the patient, unless highly myopic, is deprived of the use of the eyes for all close work, such as reading, writing, etc., and this, to the large majority of our patients, is a matter of great moment. We have no right to deprive a patient of all use of the eyes for a week if it can be avoided.

Besides, the result obtained by an examination under this condition does not represent always the actual and normal static refraction of the eye. A *paralyzed* state of a muscle is not its normal condition. There is a certain amount of *tonicity* inherent in every muscle which disappears when it is paralyzed, and it is unquestionably a varying quantity in different individuals. As a result of this, we find that when the A of young people is paralyzed there is a diminution of static refraction varying from 0.5 D to 2 D. It is the custom to refer to this as the latent hypermetropia, and so, in a certain sense, it is, but it is doubtful whether we should under ordinary circumstances, and when it is low in degree look upon it as pathological or abnormal. The result of an examination of the refraction of a large number of children, ranging in age from a few hours to 10 and 12 years, seems to point to the fact that in the human eye we have from infancy to adult life a gradual evolution from the hypermetropic to the emmetropic and myopic condition.[1] Dantel, from the results of an exam-

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[1] It is a significant fact in this direction that the eyes of most lower animals are hypermetropic—some of them very highly so.

ination of a large number of persons of all ages as to their manifest and total hypermetropia, finds that only $1/3$ of the total H is manifest from 6 to 15 years; $1/2$ from 16 to 25; $2/3-3/4$ from 26 to 35, and the two are equal only after the 36th year. He finds also, as a matter of experience, that in eyes otherwise sound a correction of manifest H is quite sufficient. With the rarest exceptions, all infants are hypermetropic, and few children of even 10 years, but show much decrease of their refraction on paralysis of the accommodation. This hypermetropia is overcome, as a rule, by the normal tonicity of the ciliary muscle, which I do not think we have a right to regard in the light of a pathological muscular spasm. When the ciliary muscle is paralyzed by a drug its natural tonicity is of course lost, and we have a correspondingly diminished refraction, but when the effect passes off, the tonicity returns and with it increased refraction, and the eye resumes its previous optical state.

That there is such a tonicity in the external muscles of the eye which is lost in paralysis, is clearly demonstrated by the exophthalmus, often very marked, which accompanies a paralysis of all the external muscles of the eye.

We have no means of measuring the amount of muscular tonicity, normal to the eye in any given case, for it is a question of physiological dynamics and we are not dealing with a constant quantity. The actual power resident in the ciliary muscle does not seem, in some cases, to bear any definite relation to the muscular power of the other parts of the body. We sometimes find it weak in strong persons, and occasionally disproportionately strong in weak persons, though, as a general rule, it participates in a general muscular debility.

It is apparent from this that, in young people particularly, we should not, as a rule, accept the refraction found under the full effect of a mydriatic as the normal optical state of the eye. When, therefore, there is ametropia present with the astigmatism which requires correction, the final glasses should not be ordered until a careful examination is made after the effect of the drug has passed away, for in the majority of cases

in young persons, the glasses which correct while under the mydriatic, over correct when the eye returns to its natural state, and at the beginning, almost without exception, the glasses giving full correction prove unsatisfactory, for one reason, among others, that the equilibrum between the external and internal muscles to which the eyes have accustomed themselves is destroyed, and it requires often a considerable time for them to become adjusted to the new order of things.

§70 As to the frequency of true spasm of the ciliary muscle, the opinions of clinicians are somewhat divided. Most of the continental authorities do not consider it at all common, and, therefore, except on rare occasions, do not follow the practice of atropinization before taking the refraction. Mauthner (*opt. Frh. d. Aug.*, p. 736), contends that the eye invariably shows its true static refraction under the ophthalmoscope. Hirschberg says (*Centralbl. f. prak. Augenheilk. June*, 1884, p. 169) that in the many thousands of cases that he has examined by the direct ophthalmoscopic method and with glasses, he has never met with a case of spasm. Furthermore, he says that he has never found the *objective* refraction different before and after atropinization, either in hypermetropia or in myopia. Landolt (*Traitè d'ophthalmol. par Wecker et Landolt* Tome 3) says he rarely has recourse to atropine for the determination of astigmatism.

In America, however, it has become a custom with quite a number to resort to atropinization as a routine practice in all cases.

It should be the aim of the ophthalmic practitioner to attain such skill in the determination of refraction that he shall have accuracy in his results at a minimum of inconvenience to his patients. The best method, it seems to me, is to obtain the best results possible by the methods already described, or such a combination of those that will be described later as may be deemed necessary, and give the glasses thus indicated for trial. If these should not prove satisfactory, we have still atropinization left. And as we study our different methods more closely and acquire by experience a greater skill in their use, we will find less and less need for the mydriatic.

§71. My own guide to the use of atropine, I find in the direct method of examination by the ophthalmoscope. (See Chap. VII.) If I find the patient to persistently refuse + glasses, and yet there is a hypermetropia manifest under the ophthalmoscope, or if, while looking at the fundus through + glasses, I see the vessels becoming alternately clear and indistinct, indicating an alternate relaxation and contraction of the ciliary muscle, I know that there is an excessive tonicity of the ciliary muscles which masks a hypermetropia, and then I usually use atropine in order to discover to what extent the tonicity reaches; not necessarily for its full neutralization, but as a guide in the selection of glasses that can probably be worn with comfort and advantage.

For the beginner, however, particularly when the case is complicated and the answers given by the patient are confusing and unreliable, and where the results by one method of examination do not correspond with those by another, an immediate paralysis of A may be the shortest way to a solution of the difficulty.

§ 72. What has been said in the foregoing paragraphs in regard to the accommodation, has reference to the contraction of the ciliary muscle *as a whole*, and to its influence on the ametropia which may accompany astigmatism. The accommodation in its entirety cannot affect the *amount* of astigmatism, though it greatly modifies its general character. It would seem however, from the reports of Drobowolski, Javal and others, that there can be a *partial* contraction of the muscle of accommodation producing a lenticnlar astigmatism, the effect of which would be either to create a new or increase an existing astigmatism, or to a greater or less extent to neutralize that of the cornea. According to Javal the latter would appear to be the most common effect, for he has found corneal astigmatism as discovered by means of the ophthalmometer, overcome by an unequal accommodation, and made manifest only on complete paralysis of the ciliary muscle. To this unequal contraction of the ciliary muscle he refers those cases of astigmatism which make their appearance in adult life and which give no

evidence of their existence in childhood. The unequal A seems to pass away with the increasing stiffness of the muscle and the hardening of the lens which are the accompaniments of advancing years.

§ 73. In treating of refraction by triaxial ellipsoids in Chap. II. it was shown in Figs. 7 and 8 that the monochromatic aberration of the cornea increased from the apex towards the periphery, and from this arises another disadvantage attendant on the use of a mydriatic. A wide pupil opens up the passage for a larger number of rays refracted nearer the periphery of the cornea, and in addition to increasing the circles of diffusion in all meridians, will in most instances also increase the difference in the refraction in the two principal meridians. As a result of this the astigmatism as determined in this condition will be apt to differ from that obtained with a pupil of normal size.

§ 74. We repeat in conclusion, therefore, that what we wish to obtain is the static refraction of the eye in its two principal meridians when the organ is in its normal condition and not when its instrinsic muscles are in a state of spasm or paralysis.

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CHAPTER VI.

Other Subjective Methods of Examination—Change in
the Form of a Point of Light—Adaptation of
Scheiner's Experiment—The Stenopaic Slit
—Modifications of Snellen's Fan
—Optometers.

§ 75. In view of the difficulties and liabilities to error pointed out in the preceding chapter, it is apparent that there would be an advantage in having at command a number of different methods to which we could appeal in case of doubt and for verification. Fortunately we are not without such resources.

§ 76. The various means used in the diagnosis of astigmatism can be divided into two general classes; the subjective and objective.

§ 77. In the *subjective* methods we rely entirely on the statement of the patient as to how the test objects appear and base our diagnosis on these alone.

§ 78. In the *objective* methods we are independent of the statements of the patient, and rely solely on our own observations.

§ 79. For errors in the first method the patient is mainly responsible. For errors in the second the observer is himself accountable.

The methods described in Chap. IV belong to the first named class, and before going on to consider those of the objective class we will give an account of other subjective methods which have been found useful.

§ 80. Changes in the Form of a Remote Point of Light.—This method, which was suggested first by Airy and was used very extensively by Donders in the beginning of his

studies of astigmatism, gives us directly a very correct idea of
the direction of the faulty meridians.   Donders' plan of using it
was as follows: In front of a window-pane of ground glass he
placed a black board about thirty-five inches square, in the
middle of which was a perforated metallic plate.   In front of
this perforation can be brought a diaphragm with openings vary-
ing in size from $1/2$ to 10 mm.   The patient is required to look
at one of these openings, having a diameter of from 2 to 4 mm.,

Fig. 22.
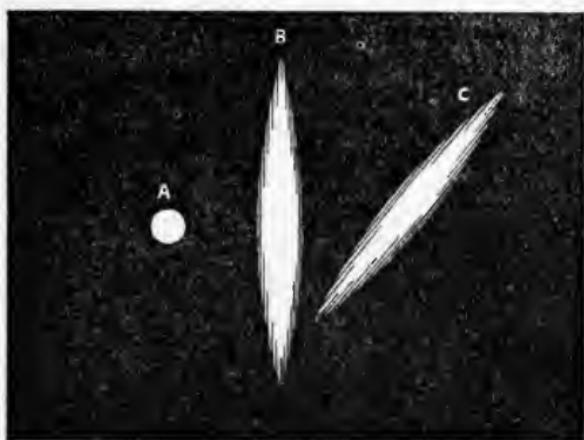
THE FORM OF A DISTANT POINT OF LIGHT AS SEEN BY AN ASTIGMATIC EYE.

at a distance of from 10 to 15 feet.   By means of $+$ and $-$
glasses, if necessary, we produce alternate M and H, when if
there be any astigmatism present the point will be observed to
be drawn out in opposite directions in the two different
conditions, indicating the meridians of greatest and least
refraction.

§ 81.   But even without the aid of the spherical lenses it is
easy to determine the direction of the meridians when astigma-
tism is present.   In simple astigmatism either myopic or
hypermetropic, and in the compound hypermetropic form
the spot of light instead of appearing round and sharply
defined, as at A, Fig. 22, will appear at a distance of
four meters drawn out, say at 90°, as at B.   In the simple

forms the ametropic meridian will correspond in direction with this, because while the rays falling in the vertical planes of the meridian at 180° are brought to a focus on the retina, those falling in the horizontal planes, in meridian 90°, unite either in front of or behind the retina and form circles of diffusion which spread the image out in an upward and downward direction. If the round spot is not seen clearly at the usual distance of twenty feet, indicating the compound myopic form, then the patient should be brought nearer until there is a distinct elongation in some direction, say at 45°, as at C, Fig. 22. We then know that this is the direction of one faulty meridian, and, of course, the other must be at right angles to it.

§ 82. But while this gives us the direction of the principal meridians, it furnishes no information as to the form of the astigmatism, the light spot being drawn out in the same direction in M and H.

As the circles of dispersion, however, are formed in a different manner, according as the retina lies in front of or behind the focus of the refracting media, we are enabled in any case to determine, in a very simple way, to which category the eye belongs.

When the spot is drawn out in a vertical direction, for example, in the case of H, the dispersion circles are homonymous, that is to say, those belonging to the upper part of the image are formed on the retina above, those belonging to the lower part of the image, below. The upper retinal impression is, under these circumstances, projected downward, and the lower one upward. In the case of M we have an opposite state of affairs; the upper rays, after crossing, form circles of dispersion on the retina below, and these are projected above, while the upper dispersion circles formed by the lower rays are projected downward.

When, therefore, a diaphragm is brought from above downward to the edge of the pupil, so as to cut off the upper rays forming the dispersion circles, the lower portion of the diffusion line disappears in case of H, and the upper part gives way first in case of M. The same principle holds good, of

course, whatever the direction in which the spot is drawn out; if, in using the cutting-off diaphragm, the end of the line opposite to the direction from which it advances disappears first, then it is H; if the dispersion circles of the same end disappear, it is M.

§ 83. Laidlaw Purves has devised a contrivance by which the inclination of the line can be determined with precision. He draws a semicircle on the screen with the round opening as a center, which is marked off in degrees. The degree to which the diffusion line points gives of course the direction of the meridian that is at fault. In order to facilitate still further this reading, he has a second screen movable behind the first, with a hole in it, which is seen through a semicircular opening in the first, made just below the semicircle bearing the degrees.

To the astigmatic eye both light spots appear drawn out in the same direction, and if the opening in the movable diaphragm be turned until its diffusion image is on a line with that of the center opening, the degree under which it stands marks the astigmatic meridian.

§ 84. But we have still no idea of the degree of the astigmatism, except that in cases where the dispersion line is long we know that it is higher than when it is shorter. For its more accurate determination we try cylindrical glasses, with their axes at right angles to the direction of the line of diffusion, and when we find one, be it + or —, which makes the light spot again round, it measures the degree and gives us the form of the astigmatism.

Snellen's and Dennett's modifications of Stokes' lens are very handy for this purpose, since, by rotating the disks, we obtain a number of different cylinders in rapid succession.

Purves also employs this same method for determining the ametropia that may be associated with astigmatism in the compound forms. This he does by finding the cylindrical glass which reduces the dispersion circles in each meridian separately. Having this, it is easy to find the ametropia common to both, and that which is in excess in one meridian.

Example :—The spot of light is generally diffused in outline, but is drawn out at 90° (vertically). A —2^{ey} axis 90° causes the lateral dispersions to disappear and gives the vertical line sharply defined edges. The horizontal meridian is, therefore, myopic 2D. With —3.5^{ey} axis 180° the spot is drawn out at 180° and has clean cut horizontal edges. M in the vertical meridian therefore = 3.5, and the case is one of comp. myopic astig.: —2 $\bigcirc$ — (3.5 —2 =) 1.5^{ey} axis 180°.

§ 85. Strawbridge, of Philadelphia, has also made a modification of this method. He has a semicircle of radiating slits like Snellens' fan cut in the diaphragm around the light spot, and these are marked in degrees. The line in the direction of which the light spot in the center is drawn out shows the inclination of the faulty meridian.

Fig. 23.
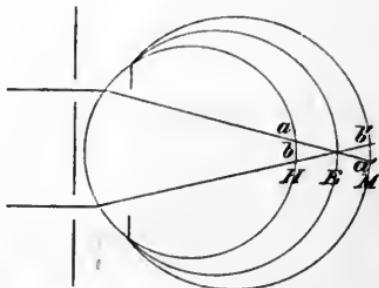
SCHEINER'S EXPERIMENT FOR DETERMINING HYPERMETROPIA AND MYOPIA.

§ 86. ADAPTATION OF SCHEINER'S EXPERIMENT.—The experiment first described by Scheiner is well known. When a small illuminated object—as a candle-flame—is looked at through two small holes in a diaphragm, placed so close together that both shall fall within the area of the pupil, only one image is pictured on the retina when it lies at the focus of the refracting surfaces of the eye, as at E, Fig. 23. If the retina, however, is found either in front of (H) or behind (M) the focus, two images are formed, and two candleflames will be seen. On the distance separating these two images and their relation to each other is based a diagnosis of the refractive condition of the eye.

When the retina lies at H, images are formed at a and b and these, when projected outward, will be *crossed* or *heteronymous*—that is, the image corresponding to the upper hole will be referred below, and that belonging to the lower opening will be projected upward. When therefore, the upper hole is covered by a card brought in front of it, the lower candle will disappear; and when the lower hole is covered, the upper image will go.

When, on the contrary, the retina lies back of the focus (M) the two images, a' and b', will be projected *homonymously*, and the upper image will correspond to the upper hole and the lower image to the lower hole; and when the light coming through the upper hole is cut off, the upper image will disappear, and the lower image will disappear when the lower hole is covered.

The existence of myopia or hypermetropia can, therefore, be determined with great readiness by simply covering one of the holes. If on covering the upper hole, for example, the lower candle disappears, there is hypermetropia; if, on the other hand, the upper candle disappears, there is myopia. The distance separating these images will furthermore give us some idea of the degree of ametropia; the greater the distance between them, the greater being the amount of myopia or hypermetropia. The ametropia, however, can be measured exactly by placing in front of the holes a $+$ or $-$ glass which fuses the two images. The number of this lens giving the degree of M or H.

§ 87. This principle can be employed in testing for astigmatism. Since the distance between the two flames varies with the degree of ametropia, when the holes are placed successively before the different meridians of the eye there will be a variation in the distance between the images in case there exists a difference in the refraction of these meridians. In turning the holes before the eye one meridian will be found where the images are farthest apart and another where they are closest together. These are the principal meridians, and the direction of the line uniting the two holes in these positions respectively, gives us the direction of these meridians.

We determine the form of the astigmatism in the same manner as we do that of the general ametropia, by observing which flame disappears when one of the holes is covered. If the flame on the same side disappears, it is myopia, if the opposite one goes, it is hypermetropia. The degree of astigmatism is expressed by the cylindrical lens which, when its refracting surface corresponds to the faulty meridian, makes the distance between the images the same when the holes are brought before all the meridians.

§ 88. Thomson, of Philadelphia, has devised a plan and invented an apparatus based on these phenomena, for the diagnosis of the various forms of ametropia without the aid of glasses.

For this purpose he employs two small flames whose distance from each other can be varied. To the emmetropic eye these two flames appear sharply defined in outline, and are united only when one passes behind the other. With the ametropic observer, however, the case is different. To him each flame is an area of diffused light, whose extent is proportioned to the degree of his ametropia. In making an examination, the patient is placed at a distance of 5 meters from the two flames and they are gradually approached until the edges of these areas of diffusion touch. Thompson has calculated the size of these diffusion areas for all degrees of M and H for the distance of 5 meters, and you read off on the arm of his instrument the amount of departure from the emmetropic condition. In the instrument he has described each space of 2.5 cm. corresponds to one dioptry of refraction.

While the *degree* of the ametropia is thus arrived at with tolerable precision you have not as yet the *form*. This you obtain by passing a card partially across the pupil, when, as explained in the preceding paragraph, one side of the diffusion areas of both flames will be cut off. On the principle we have already explained, if the cutting off is homonymous it is a case of M, if heteronymous it is H.

In making use of these diffusion images for the diagnosis of astigmatism, advantage is taken of the modification in the form

of the area of diffusion, as in Donders' method, and of the ability by means of his instrument to place the flames in a line corresponding to any chosen meridian.

In simple M or H the diffusion areas are round, and when their edges are in contact they remain so in whatever meridian they may be brought, by a movement of the bar of the instrument.

In astigmatism, on the other hand, the diffusion areas are no longer round, but oval, and the two do not remain in contact when the arm is placed in all meridians. The direction in which the elongation takes place shows the inclination of the principal meridians. The arm is moved until the blurred images are in contact at their shortest diameters and its inclination to the horizontal is noted, and the amount of ametropia in that meridian read off on the scale. The bar is then turned at right angles to this and the long axes of the images are brought in contact. The position of the movable flame on the scale gives the ametropia in this meridian, and the difference between the two gives, of course, the amount of astigmatism.

§ 89. EXAMINATION WITH THE STENOPAÏC SLIT.—When a diaphragm, having a slit a millimeter in diameter, is rotated before an astigmatic eye looking at test objects, placed at a distance of 5 or 6 meters, it will be found that when the slit corresponds to one certain meridian, vision is best, and when it is at the meridian at right angles to this it is worst. This gives us the direction of the principal meridians, and we can easily find the refractive condition of each separately. When the slit is at 90°, for instance, all the rays which would pass through the other meridians are cut off by the diaphragm, and by testing in the usual manner with + and — spherical glasses we can find with considerable exactness the state of refraction of the meridian open to the passage of rays ; and so for any other meridian whose refraction we may wish to determine.

Example: $V = \frac{4}{36}$. When the slit is at 130° $V = \frac{4}{12}$; when at 50° it is less than $\frac{4}{60}$.

The slit being at 130°, on trying, in the usual manner as de-

tailed in Chap. IV., various spherical glasses, we find that with
— 1.5 V = ⁴/₅; therefore meridian 130° has M = 1.5.   Turn-
ing it now to 50° we find that it takes —4.5 to bring V = ⁴/₅;

Fig. 24.
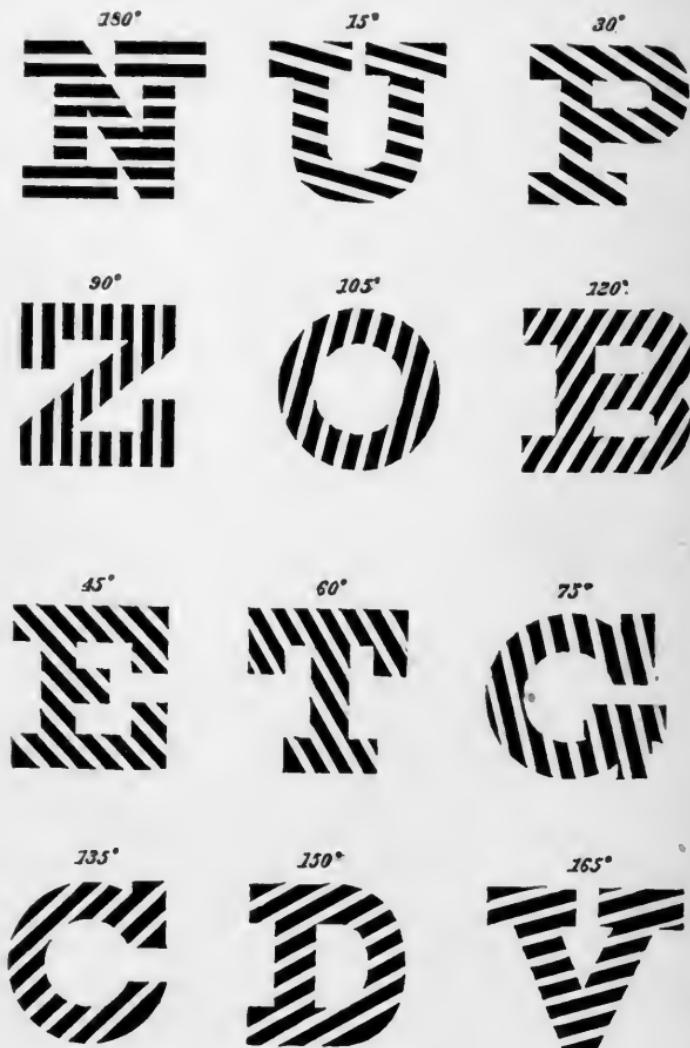
TEST TYPES BY PRAY.

hence 50° has M = 4.5. The *amount* of astigmatism is there-
fore 4.5 —1.5 = 3: But with this there is combined a myopia,
which is common to both meridians, of 1.5. The case is one

of compound myopic astigmatism : — 1.5 ◯ — 3 axis 130°. The disadvantage of this method is that visual acuteness is considerably reduced by the exclusion of so much light by the diaphragm, and by the circles of dispersion, small it is true, but still appreciable, which come from the few rays which pass through the slit in the other meridian.

§ 90.   THE TEST LETTERS OF PRAY.—A very useful and convenient modification of the principle of Snellen's fan are the test letters of Orestes Pray.

These letters are formed by a series of parallel lines which are inclined, for each separate letter, at a different angle to the horizontal, as shown, reduced, in Fig. 24. To the astigmatic eye the lines composing one of the letters, as P for example, will appear most distinct in outline, while those of another, as B, running at right angles to these, will appear most indistinct. The patient, being placed at such a distance that the letters as a whole are distinguished (20 feet), is able to tell us which letters these are, and we, knowing the inclination of the lines in each, are at once informed as to the direction of the principal meridians—remembering always that the most and least distinct lines are at right angles to their respective meridians.

§ 91.   John Green, of St. Louis, devised a set of lines drawn from a common center on a clock face, as in Fig. 25. The patient indicates the lines which are most distinct by the figure on the clock with which they correspond, and the examination is conducted in the same way as when made with Snellen's fan. He has also designed many other figures, but they are all modifications of this general plan, and are used in the same manner.

§ 92.   Oliver, of Philadelphia, has devised a modification of these methods which consists of a disk with three concentric series of radiating lines corresponding in size with three visual angles. Two of Pray's letters are attached to a rotating disk-perimeter, one at an angle of 90°, the other at 180°. The patient must designate which of the radiating lines he sees most distinctly. The disk is then rotated until the lines of one of the letters appears most distinct, which will mark the direction of

one principal meridian; the other principal meridian will of course be at right angles to it.   The examination is then conducted as in the usual way with Snellen's fan.

§ 93.   Culbertson has used the doubling of an object when looked at through two prisms with their bases together as a

Fig. 25.
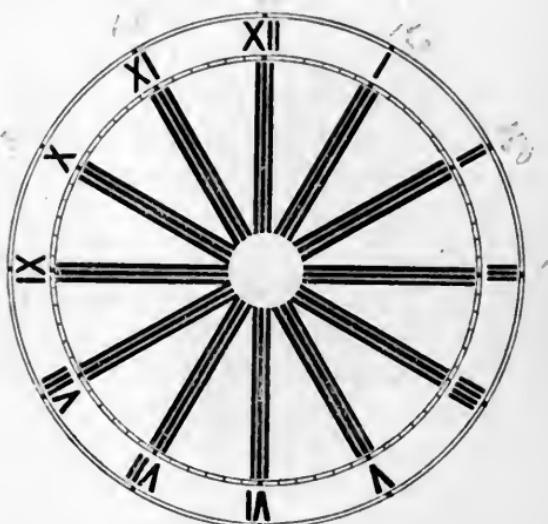
THE CLOCK FACE OF GREEN.

means of diagnosing ametropia.   When the object is of a definite size and at a certain distance the images touch at their opposite edges.   The degree of ametropia is determined by the glass, which when placed before the eye gives this touching for the standard size and distance of the object.   The refraction of the separate meridians can be taken by this means as well as that of the eye as a whole.

§ 94.   Nearly all of the various *optometers* can be used for the determination of astigmatism.   An objection to their use, however, is that with them it is even more difficult than in the examination with test-objects at a distance to control the accommodation; and to obtain anything like accurate results it is necessary to paralyze the ciliary muscle.

§ 95.   The apparatus of *Bravais*, of Lyon, since it employs a

principle somewhat different from the others, merits a short description. It consists of a tube of convenient length, at one end of which is a convex lens with a focal distance shorter than the length of the tube. The other end of the tube is closed with a diaphragm having a small round opening in its center. The patient looks through the tube with the hole turned toward a lamp or window, and if astigmatism be present the round light spot will appear oval as in Donder's experiment (§ 81) with its long diameter in the direction of the most strongly refracting meridian. In the interior of the tube there is another spherical lens which can be rotated on its axis in any desired meridian, and thus made to act as a cylinder (see § 25). An index on the outside shows the degree of inclination of this lens when the light spot again becomes round, and a table shows the number of the cylinder to which this inclination corresponds.

§ 96. Zehender has adapted the method first employed by Young into the form of an instrument which he uses for estimating the degree of astigmatism and the direction of the principal meridians. He has a few fine threads stretched parallel to each other across one end of each of two tubes, one of which is inserted, telescope fashion, into the other. When the tubes are so placed that the threads of one lie close against and at right angles to the other, a non-astigmatic eye sees them all with equal distinctness when the tubes are rotated to any position on their common axis. When the inner tube is withdrawn, separating the two bands of threads to any considerable distance, however, it is no longer possible for the eye not astigmatic to see both bands distinctly at the same time. When the eye is astigmatic this, of course, is not always true. When the bands are at right angles and the tubes are revolved on their long axis there is one position in which the threads of one band are seen clearest and those in the other least so. The bands then lie in the direction of the principal meridians. Spherical glasses are now placed before the eye, one after another, until one is found which brings out the indistinct band clearly. The number of this glass expresses the amount of astigma-

tism in the meridian at right angles to the direction of the
band.

§ 97. *Javal's Astigmometer:*—As stated in § 94, one of
the principal defects inherent in all the methods of opto-
metric measurement where the test object is within a
finite distance and close to the eye, is the almost unavoidable
tendency to the use of the accommodation. We have always
been accustomed to use the accommodation when the object
is near us and when we know that it is at the other end of a
short tube we instinctively accommodate for that distance.

When the visual axes are parallel, however, as when we look
at infinity, the ciliary muscle is usually relaxed. Javal has
taken advantage of this fact in the construction of his astigmo-
meter.

As a test object he uses a circle with lines drawn through its
center at every 15 degrees. This circle is looked at through
a convex lens having a focus of about five inches. To the eye
free from astigmatism, of course, all the lines appear of the
same distinctness. The astigmatic eye sees some more clearly
than others, and the card on which the circle is drawn is moved
back and forth until one line becomes sharply defined. This
line will lie in the direction of the meridian of greatest refrac-
tion. Concave cylindrical lenses are then placed successively
before the eye, beginning with the weakest, with their axes at
right angles to the distinct line until one is found through
which all the lines appear with uniform sharpness of outline.
We thus obtain the direction of the principal meridians and
the degree of astigmatism. The ametropia that may be asso-
ciated with the astigmatism is determined subsequently. In
order to avoid any interference on the part of the accommoda-
tion, he has a second circle drawn on the card, with its center
at the same distance from the center of the circle with the lines
as separates the visual axes of the eyes in a state of parallelism.
When the two eyes look at these circles, therefore, there will
be two circles seen unless the visual axes are parallel. When
the patient sees only one circle with both eyes open, the eyes
are adapted for distant vision and the accommodation is re-
laxed.

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CHAPTER VII.

Objective Methods of Examination—The Ophthalmoscope as a Means of Diagnosis in Astigmatism.

§ 98. In the objective methods of examining for astigmatism we do not rely, as in the subjective methods just described, on the statements of the patients for the data of diagnosis, but trust entirely to our own observations. The advantage of the plan is very great, since it makes us independent of the patient, and eliminates the most potent sources of error. For a satisfactory examination by the subjective methods a certain—in fact a very considerable—amount of intelligence on the part of the patient is necessary. This, unfortunately, on account of age or mental condition, we do not always have at command, and had we not some other means of getting at the facts it would be often impossible to arrive at any intelligent or definite idea as to the refractive state of the eye.

The special advantages, as well as the defects and shortcomings of the various objective modes of examination, will be pointed out when we come to a consideration of each plan in detail.

§ 99. The Ophthalmoscope in the Diagnosis of Astigmatism.—When Helmholtz invented the ophthalmoscope he not only gave to the profession an instrument by means of which the condition of the interior of the eye could be examined in minutest detail, but, what is hardly less valuable, put into their hands an apparatus for testing its optical state.

The ophthalmoscpe, as an optometer, has now become one of our most important and reliable means of diagnosis.

§ 100. There are two ways of making an examination with the ophthalmoscope. One is the *direct* method, giving an

erect and virtual image of the fundus of the eye; the other is the *indirect* method, in which the image of the fundus is *inverted* and actual. The manner in which these two kinds of images are obtained is essentially different in the two methods.

For the fundamental principles of ophthalmoscopy we shall have to refer the reader to general treatises on the subject.

§ 101. *The Direct Method of Examination.*—In this method we look directly into the eye to be examined and see the fundus in its proper position, but under the magnifying power of its refracting media.

It will simplify our study somewhat if we remember that when we employ the ophthalmoscope as an optometer in ametropia, we are only using one optical instrument to neutralize the action of another, in the same way as we determine the strength of a + lens by the — lens necessary to overcome its refraction and reduce its optical properties to zero.

As in the determination of general ametropia by glasses, it is the object to so alter the course of parallel rays that they may appear to come from the far point of the eye under examination, so, in making a diagnosis with the ophthalmoscope by the direct method, it is the purpose to render the rays which come out of the ametropic eye, and are directed to its far point, parallel. In both cases the glass which does this gives the amount of deviation of the eye from the emmetropic condition.

§ 102. This principle, as applied to the direct method of ophthalmoscopy, is shown in Fig. 26. The eye V is myopic, and the rays *e f* coming from a point *o* of the illuminated fundus, after passing out of the eye converge towards its far-point situated at *a*, six inches in front of the cornea. The eye E of the emmetropic observer is not adapted for converging rays, and in order to have the rays *e f* brought to a focus on his retina they must be rendered parallel. This can be done by placing a concave lens having a negative focus of five inches behind the ophthalmoscopic mirror and one inch in front of the eye to be examined. The rays *e f* directed to the negative focus of the lens, also at *a*, will then be made parallel,

and the emmetropic eye E being in a state of repose and adapted for parallel rays, will unite them on its retina, and they will form there a clear and distinct image of the point o. This same lens, acting in a contrary sense, will render parallel rays coming from distant objects divergent as though they

Fig. 26.
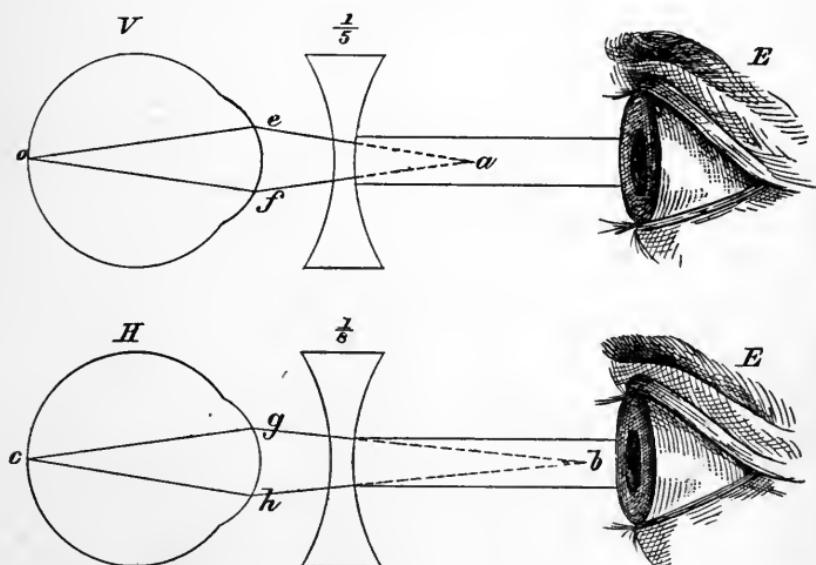
DIRECT METHOD OF EXAMINATION WITH THE OPHTHALMOSCOPE.—Alteration in the course of the rays by concave lenses in the principal meridians in compound myopic astigmatism.

came from the point a, five inches in front of it (that is the far-point of the eye). Therefore, the same concave lens through which an emmetropic observer can see the fundus of a myopic eye distinctly expresses the degree of myopia of the observed eye, and will adapt it to vision at an infinite distance.

In the example we have taken the rays $e\,f$ belong to one meridian—the vertical—but if the refracting surfaces are the same in all meridians, as in ordinary simple myopia, the law applies to the others as well, and through all points situated in every meridian there will proceed rays convergent towards a which which will be rendered parallel by the same lens $(-^1/_5)$ and be again united on the retina of E. Under these conditions,

therefore, the emmetropic observer E will be able to see with distinctness all the details at the fundus of V, and the fine retinal vessels running in one direction will be made out as clearly as those running in any other.

§ 103.   The state of affairs is very different, however, when we come to deal with an eye having a different curvature in each meridian, as in regular astigmatism.

Here the far-point to which the emerging rays converge, in the case of compound myopic astigmatism, will be in a different position for the two principal meridians, being nearest to the eye in the most strongly refracting, and farthest from it in the weakest.   Letting V and H, Fig. 26, represent the two meridians of greatest and least refraction respectively, a will be the far point of V, 6 inches in front of the cornea, and b that of H, 9 inches in front of the cornea.   It would not be possible, therefore, for any one spherical lens to render the rays coming through these two meridians parallel so that they would be united at the same place on the retina of E, and as a consequence the observer could never obtain, by such a lens, a uniformly distinct image of the details of the fundus.   If the delicate vessels running in the vertical direction were seen sharply with — $1/8$, those running in the horizontal direction and corresponding to the opposite meridian, V, would be blurred.   When, on the other hand, a — $1/5$ lens is used which renders the fine vessels, running horizontally and corresponding to the vertical meridian V, sharply defined, those running in the opposite direction are indistinct.

§ 104.   On this principle and in consonance with these facts is based one of the methods of diagnosing astigmatism by means of the ophthalmoscope.

The refraction of each principal meridian is taken separately, just as if it were a case of simple ametropia.   The mirror of a refraction ophthalmoscope is brought close to the observed eye and the disk containing the correcting lenses is turned until one lens is found through which the finest retinal vessels running in one direction appear sharply defined.   Let us say, for example, that the delicate vessels running horizontally over

the outer edge of the optic disk are clearly made out with — 3.
The fine vessels running almost vertically in the region of the
macula lutea will, through the same lens appear blurred and in-
distinct (the accommodation of the observer remaining, of
course, in a state of complete relaxation).   The converging
rays coming from these horizontal vessels and passing through
the vertical meridian of the eye are rendered parallel by the
— 3 and thus become adapted to the emmetropic observer.
The vertical meridian has therefore M = 3.   The small verti-
cal vessels near the macula are seen distinctly with — 1, while,
with the same lens, the horizontal vessels at the edge of the
disc appear blurred.   The horizontal meridian which refracts
the rays coming from these vertical vessels is myopic to the
extent of 1 D.   The case is one of Comp. My. Astig. — 1 $\bigcirc$
— 2 axis 180°.   The same principle applies, of course, to all
the other forms of astigmatism.   The direction of the vessels
which are seen most distinctly gives the direction of the prin-
cipal meridians, and the lenses +,or —,through which they are
thus seen gives the degree of ametropia for each respectively.
The degree of astigmatism is expressed by the difference in
the refraction of the opposing meridians.

§ 105.   The defects of this method are two.   There is, first,
the impossibility of determining in all cases the exact direction
of the faulty meridians.   The fine vessels of the retina which
we employ as test-objects do not spread out regularly in all di-
rections like a Snellen's fan, and frequently there is no one
which follows the exact line of the axis of either one of the
principal meridians.   Our estimate of the direction of the me-
ridians is, therefore, only approximative, and cannot be made
certain, except by some one of the other methods.   Neverthe-
less, we can always obtain an idea of their general direction,
and this is frequently of the utmost importance as furnishing a
key to the situation.

The second defect applies equally to the determination
of general ametropia by this means, and that is the want of ex-
actness in estimating the degree of ametropia in each meridian.
My observations and experience as a teacher have convineed

me, that it is not given to all men to be good ophthalmoscopists. particularly by the direct method.   A perfect result in determining refraction in this way requires, in the first place, an absolute control over the accommodation on the part of the observer, and this cannot be acquired by every one.   A very slight change, one of which we are not at all conscious, in the tension of the ciliary muscle will make a difference of 0.5 or 0.75 D.   Then, again, if the media are not quite clear and the pupil is smâll, as we have it nearly always in advanced life, it is not possible to distingulsh within 0.75 D, between the lenses which give the clearest outline to a particular vessel.   It is to be remembered, also, that particularly in M, the vessels which serve us as test-objects are not all on the same level.   Those at the edge of the disc are in advance of those at the macula, and it is the refraction of the meridians at the latter place we want. Where the pupil is so small as to seriously diminish the illumination of the fundus, a mydriatic should be used.   Of these, hydrochlorate of cocaine and hydrochlorate of homatropine are best, since their effect on the accommodation is more transient than that of atropine.

· While acknowledging that there are some brilliant exceptions to the general rule, I yet believe that, taking ophthalmoscopic observers in the mass, it is not possible in an average of cases, to détermine within 0.75 D of the actual refractive condition of the eye, by means of the ophthalmoscope.

But notwithstanding this, ophthalmoscopic examination by the direct method is, and will probably always remain, one of our readiest and most reliable aids in the diagnosis of astigmatism ; and all students of ophthalmoscopy should diligently practice the method and endeavor to acquire the greatest amount of skill possible in its use.

§ 106.   The general appearance of the fundus of the astigmatic eye is very striking, particularly in the high degrees of the anomaly and, when once seen and understood, is not likely thereafter to be mistaken for anything else.   With the exception of keratometry, to be considered later, I know of no other method which gives us so promptly such important informa-

tion as to the existence of these high degrees of astigmatism which are so puzzling when tested by the subjective methods alone. A single glance is often sufficient to reveal to us the direction of the principal meridians, and furnish an indication as to the special form of the anomaly.

Fig. 27.
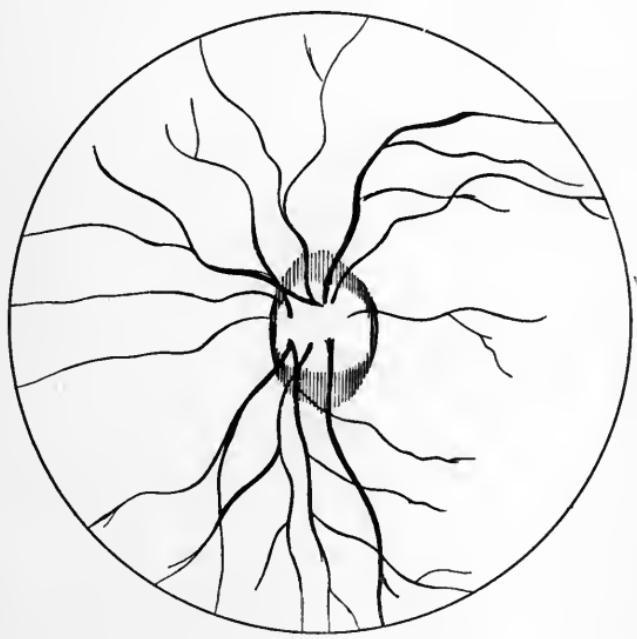
DIAGRAM REPRESENTING JAEGER'S INACCURATE DRAWING OF THE FUNDUS OF THE ASTIGMATIC EYE AS SEEN BY THE DIRECT METHOD.

There does not exist, to my knowledge, a drawing representing the fundus of the astigmatic eye with any approach to faithfulness. Jäger has in his well-known atlas (Fig. 31 of the smaller edition), a drawing which purports to give the appearance of the fundus as seen in the direct method, but it is far from a true picture. That so faithful a delineator as Jäger should commit such an error is indeed wonderful.[1] Fig. 27 is a diagrammatic

[1] In Loring's text-book of ophthalmoscopy which has appeared since the MS. of this chapter has been in the hands of the printer, there is a representation of the fundus of the astigmatic eye. While more nearly accurate than Jäger's, it yet fails to give that marked contrast which exists between the distinctness of the vertical and horizontal vessels.

copy of this drawing and a glance at it will show that it could not by any possibility be the fundus of an eye as seen through an astigmatic refracting system. The fine retinal vessels are seen running in all directions with equal distinctness which, as we have shown, could not be the case in an astigmatic eye. The only part of the drawing which has any semblance of truth is the appearance of the optic disk. This does give a very good picture of the disk as it appears when the meridian is adapted to the eye of the observer. The vertical outlines are sharp, while the horizontal outlines are blurred and indistinct. But under the optical conditions producing this effect it would not be possible to see the delicate retinal vessels running horizontally over the edge of the disk as they are represented in Jäger's figure. So far as I know attention has not before been called to the falseness of this representation. Even so good and able an observer as Nettleship has copied it into his textbook without any allusion to its inaccuracy.

I have endeavored to give in Fig. 28 a more nearly accurate representation of the appearance of the fundus of an astigmatic eye when examined by the direct method. It was sketched from a case of simple myopic astigmatism of 6 D axis 180°, and it is given as seen without any correcting lens.

The horizontal meridian being emmetropic, and the eye of the observer being emmetropic also, the parallel rays which pass out through this meridian are focused properly on the retina of the observer, and consequently all vertical outlines are distinct. The large retinal vessels passing upward and downward over the face of the disk are seen sharply defined with the light reflex along their center very clear. As soon, however, as the vessels become deflected to one side and assume an oblique or horizontal direction out of the axis of the emmetropic meridian their outlines become blurred, the reflex is lost, and their course is marked only by an ill-defined band whose outlines merge into the surrounding red of the fundus. Wherever in their course, they again assume a vertical direction, their outlines become once more sharply defined; all light points are converted into vertical light lines, and all black

points or masses become vertical black lines, giving the fundus a striking appearance of vertical "streakiness." The vertical sides of the optic disk are sharp in outline while superiorly and inferiorly they are "fuzzy" and indistinct, and, as a whole, the disk appears vertically elongated. Such an ap-

Fig. 28.
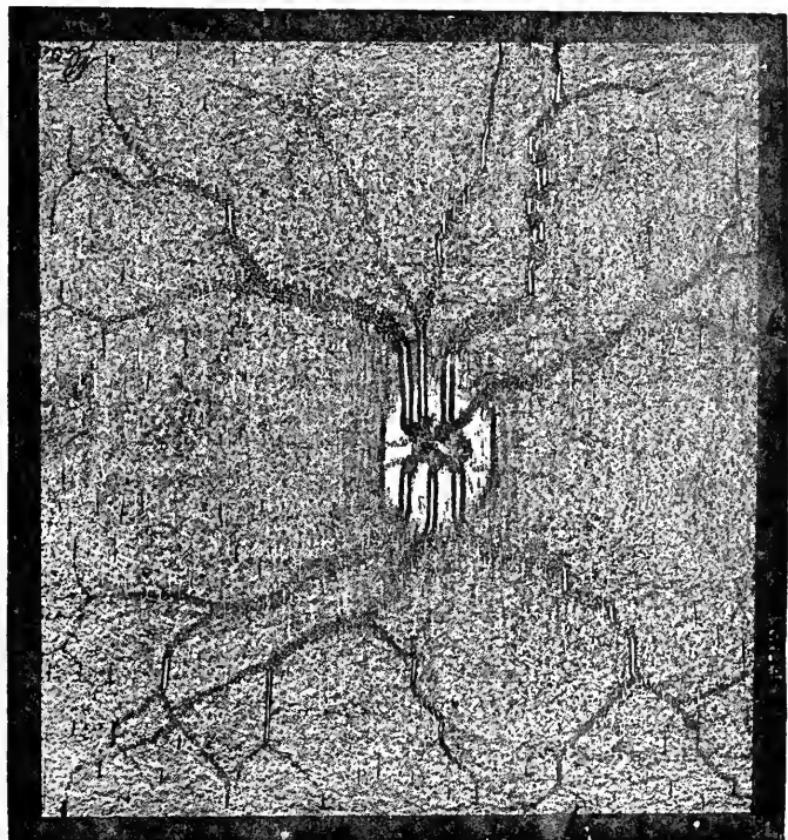
APPEARANCE OF THE FUNDUS OF AN ASTIGMATIC EYE AS SEEN BY THE DIRECT METHOD OF EXAMINATION.

pearance could easily be mistaken by a novice for a pathological condition—a retinitis or neuro-retinitis. I distinctly remember in my first workings with the ophthalmoscope, before I had studied astigmatism, making such an error in diagnosis, which was, however, promptly corrected by one whose experience in such matters was greater than mine.

A very good idea of these appearnces may be obtained by looking at a drawing of a normal fundus in any ophthalmological atlas through a + cylindrical lens of six inches focus placed with its axis vertical, close to the eye and at six inches from the drawing. If the accommodation of the observer is then relaxed the rays coming from the vertical vessels and passing through the horizontal meridian will be rendered parallel by the lens, focused by the observing eye on its retina, and seen clearly, while the others, coming from the horizontal and oblique lines and passing through the other meridians will unite, if at all, behind the retina, and give images with blurred outlines.

§ 107. The same principles apply, of course, to every form of astigmatism with the principal meridians lying in all possible directions. If, in the case taken as an example, the myopia of the vertical meridian is corrected by a — 6 spherical behind the ophthalmoscopic mirror, rendering the horizontal meridian hypermetropic, the horizontal vessels will come out clear and distinct, and all the vessels running vertically will appear blurred, since the rays coming from them cross behind the retina, while the disk will seem to be drawn out horizontally, with its superior and inferior borders sharply defined. If the meridians are oblique, the disk will be elongated in corresponding directions when the ametropia of each is corrected, and those portions of the vessels which run in these directions will be sharply outlined.

§ 108. In examining for ametropia by the direct method care should be taken to so arrange the relative position of the head of the patient to the light that the ophthalmoscope shall be as nearly as possible at right angles to the optical axis of the eye under examination. If this precaution be not taken and the correcting glass behind the mirror is inclined to the direction of the rays coming from the eye, there will be a cylindrical action on the part of the lens, as demonstrated in § 25. It was for the purpose of obviating this source of error, among others, that the various "tilting" ophthalmoscopic mirrors were invented, (Loring, Wadsworth, DeWecker, and others).

§ 109.   Dennett, Uhtoff and Parent have described modifi-
cations of the refraction ophthalmoscope by which it is possi-
ble to examine the fundus of the astigmatic eye through cor-
recting cylinders.

Uhtoff's (Shoeler's) instrument consists of a disk with ten
cylinders (+ and —) which is fixed to one side of the instru-
ment back of the mirror, the disk containing the sphericals
being fastened at the other side.   The glasses contained in
these disks can be brought by rotation, as in the ordinary in-
struments, behind the perforation in the mirror, superposed if
necessary.   The disk containing the cylinders can, in addition,
be so turned, as a whole, that the axis of the cylinder behind
the perforation can be placed in any desired direction.

Parent's instrument is constructed on the same principle.

Dennett used a small Stokes' lens behind the opening in
the mirror.   He tells me, however, that he has ceased to use
it for some time.

Such instruments I do not consider of any great value for
making a first diagnosis of astigmatism, for there are many
other means easier and more ready and reliable.   It is, how-
ever, often of great importance to be able to examine in detail
the fundus of an astigmatic eye, to determine the presence or
absence of pathological changes there.   In an eye even mod-
erately astigmatic this is not possible by the direct method.
Some means by which this can be done is, therefore, im-
perative.

The instruments above mentioned would enable us to do
this, but they are more or less cumbersome and somewhat
expensive.   It occurred to me, therefore, that some modifica-
tion of the ordinary ophthalmoscope was possible which would
enable us to use for this purpose the cylindrical glasses in our
cases of test lenses.   Such a modification I have devised, and
a back view of it is represented in Fig. 29.   It consists in
the addition of a clip, behind the disk holding the lenses, into
which a cylindrical lens can be placed.   This clip moves in-
dependently of the large disk, and has attached to it a section
of another disk holding two lenses, + 3.5 and — 3.5, which

can be brought behind the sight-hole in the mirror when needed. The degree of inclination of the axis of the cylinder is read off on a scale marked at the edge of the back of the mirror.

Fig. 29.
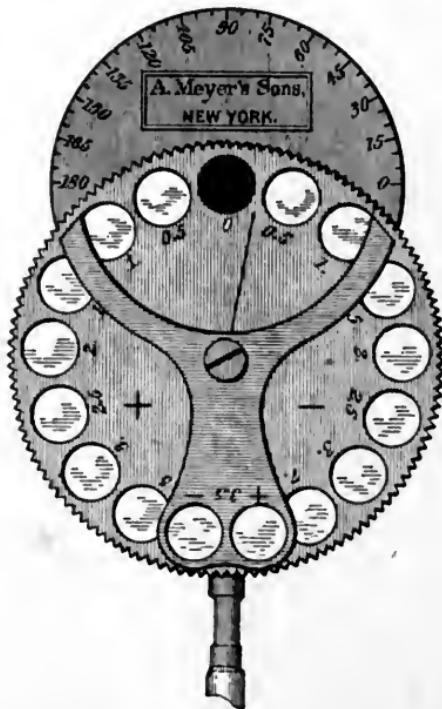
REFRACTION OPHTHALMOSCOPE WITH A CLIP FOR THE INSERTION OF CYLINDRICAL LENSES.

Though the instrument may not be of great value for the first determination of astigmatism, it is very useful for verifying the findings by other methods, and I believe will be found very convenient for that purpose, after some practice in its use.

The instrument has, too, other advantages which I think will commend themselves for its general use. By the use of the two lenses in the superposed segment a large number of lenses are obtained for the determination of general ametropia. We can get in all, fifteen plus and seventeen minus lenses, as follows: + 0.5; 1; 1.5; 2; 2.5; 3; 3.5; 4; 4.5; 5; 5.5; 6; 6.5; 7.5; 11; and — 0.5; 1; 1.5; 2; 2.5; 3; 3.5; 4; 4.5; 5; 5.5; 6; 6.5;

7.5 ; 9.5 ; 11 ; 13.   These are all that are ever needed in prac-
tice, and the interval between the numbers is sufficiently small
for the finest diagnosis.   Compared with other instruments of
good workmanship and equal usefulness the price is very
moderate [1].

§ 110.   *Examination by means of the inverted ophthalmoscopic
image.*   As already stated, the indirect method of ophthalmo-
scopic examination differs essentially from the direct method.
By the latter we look immediately upon the illuminated fundus
itself, whereas with the former, we see its actual inverted im-
age formed in the air by an auxiliary lens placed in the path
of the emerging rays.

Fig. 30.
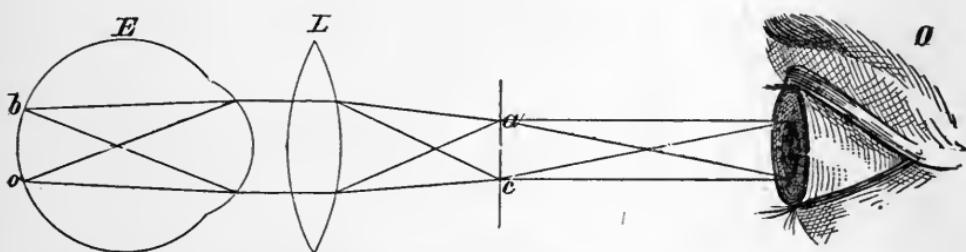
FORMATION OF THE ACTUAL INVERTED IMAGE IN THE INDIRECT METHOD OF.
OPHTHALMOSCOPIC EXAMINATION.

The size and position of this aerial image varies with
the optical condition of the eye from which the rays proceed,
the strength of the auxiliary lens used, and under certain cir-
cumstances on the distance of this lens from the cornea.   The
problems in connection with this method of observation are
therefore, somewhat more complicated than in the direct
method.   They are easily solved, however, when we bring to
our aid a few of the fundamental principles of optics.

Let us examine briefly into the manner in which this inverted
aerial image is formed.   In Fig. 30 *E* is an *emmetropic* eye

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[1] The instrument is made by A. Meyer's Sons, 93 William street, New York.   Its
price is $17.00.   The clip behind the mirror can be made by them to fit the cylin-
ders of any particular trial-case.

whose fundus is already illuminated. From the points *o*
and *b* there proceed rays which, in passing out of the eye, be-
come parallel. These rays, falling on the convex lens *L.*, hav-
ing a focal distance of 3 inches, are brought together and form
at the focus of the lens an inverted image, *a c*, of *o b*. The eye
of an observer at *O* will see this image *a c* clearly when it is
accommodated for vision at that distance. The power of the
lens *L* governs the size of the image *a c* and its position in re-
spect to *L*.

§ 111. When the eye is emmetropic, *a c* is always found at
the focus of *L*, no matter at what distance from the observer's
eyes the auxiliary lens is held, because the rays come from the
eye parallel and remain so indefinitely, and will consequently
always fall thus on the lens at whatever distance from the eye
they may strike it. If a $+\frac{1}{3}$ is used, the image will always be
3 inches in front of it: if a $+\frac{1}{2}$ is employed, it will be 2
inches, and so on.

This being the case, with the same lens the size of the im-
age *a c* must always be the same in emmetropia, at whatever
distance from *E* the lens is held.

This is true of the *actual* size of the image, but not strictly
so as to its *apparent* size to the observer at *O*. The effect of
removing *L* away from *E* would be to cause an approximation
of the image *a c* to *O*. As a consequence of this it will be
seen under a constantly increasing visual angle and as it is seen
with only one eye and is always referred to the same position
in space, its apparent size increases as it is brought closer to *O*.
It is not strictly true, then, as stated by many authorities in
ophthalmoscopy, that in emmetropia the size of the inverted
image remains unchanged in all positions of the auxiliary lens.

§ 112. In *myopic* and *hypermetropic* eyes we have an entire-
ly different set of conditions to deal with. The rays coming
from the illuminated fundus of these eyes do not emerge in
a state of parallelism, but are either convergent or divergent.

The size and position of the image produced by the auxil-
iary lens, under these circumstances, must be determined by
the laws of conjugate foci.

By applying these laws to the two abnormal optical conditions of myopia and hypermetropia we can know the position and relative size of the image to its object in any given position of the auxiliary lens on the common optical axis.

§ 113. Let us take the case of *hypermetropia* first. Here the rays emerge divergent, and those coming from the point *o*, Fig. 31,


Fig. 31.
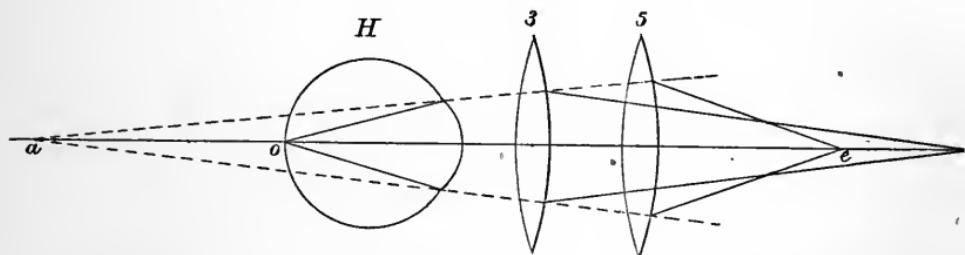
CHANGE IN THE SIZE AND POSITION OF THE REAL IMAGE OF THE HYPERMETROPIC EYE IN THE INDIRECT METHOD OF OPHTHALMOSCOPIC EXAMINATION.

after passing out of the eye assume a direction as though they came from a point *a* behind the eye, which is the virtual and erect image of *o* formed by its refracting media. This point *a* is the far point of the eye, and in this case negative. The point *a*, therefore, and not *o*, becomes one of the two conjugate foci and gives the position of the object, whose real inverted aerial image is to be formed by the auxiliary lens placed in front of the eye. When this lens is at 3, (3 inches in front of the cornea) the image of the object at *a* will be formed at *f*, a distance greater than the focus of the lens, When it is removed two inches farther to 5, the image will be formed at *e*, since in bi-convex lenses the object and the image are displaced in the same direction, and being closer to the lens will be smaller. As the lens is still further removed from *a* the image will be brought closer to the lens and diminish correspondingly in size until it reaches a point where the emergent rays become practically parallel, when the image will be formed at the focus of the lens, and will then remain of the same size, even though the lens were removed to infinity.

§ 114. In *myopia* the conditions are the reverse of those in

hypermetropia, and there is a corresponding change in the effect on the position and size of the image formed by the auxiliary lens. Here again we can assume, for the sake of uniformity, that the object is not at the point o (Fig. 32) from which the rays emanate, but at the far point a, where the emerg-

Fig. 32.
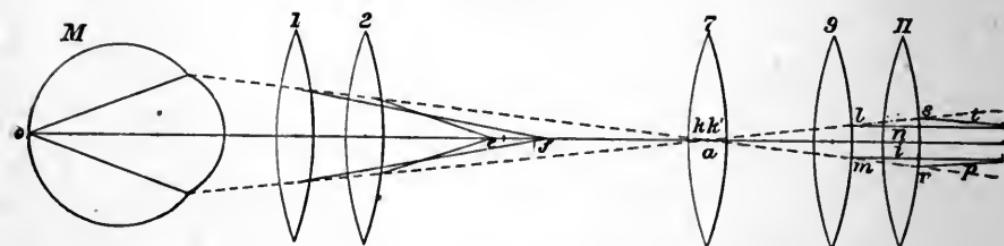
SHOWING HOW THE REAL IMAGE OF THE MYOPIC EYE VARIES IN SIZE ON REMOVAL OF THE AUXILIARY LENS IN THE INDIRECT METHOD OF OPHTHALMOSCOPIC EXAMINATION.

ing rays meet and form a real and inverted image of the object at o. The image, formed by the auxilliary lens placed in the path of these rays, will be real and on the same side of the lens as the object, when the lens is found at any place between a and the eye M. When the lens is at 1 (1 inch in front of M) the image which is seen by the observer will be at f, nearer the lens and smaller than the object at a. When the lens is advanced towards the object and is found at 2, according to the law of conjugate foci, the image advances toward the lens, is found at e, and is increased in size. As the lens is still further removed towards a, the image gets nearer and nearer the lens and increases more and more in size, but still remaining real, until it is found at 7, and occupies the same position as the object. The image and object will then be of the same size, one lying at the first nodal point of the lens, k, the other at the second nodal point, k'. On a still further removal of the lens a is left between it and the eye, the image becomes virtual, is found between the lens and the eye M, and is larger than the object at a. It continues to increase in size on further removal

of the lens until the lens is found at 9, when the object lies at its focal distance. The rays will then emerge from the lens parallel, *l n*, *m i*, and the image will lie at infinity and be infinitely large as compared with the object.

When the lens gets beyond its focal distance from the object at *a*, as at 11, the rays *s t*, *r p* emerge from it convergent, and will form, somewhere beyond, an inverted and real image of the object at *a*, being a real and erect image of *o*. As the lens is still further removed this real image approaches the lens and becomes smaller.

§ 115. This variation in the size of the real image of the myopic eye on removal of the auxiliary lens, can be demonstrated by taking, as one of the conjugate foci, the actual object at *o*; but in that case we should have to do it with a constantly varying compound optical system of the eye and the lens, making the problem much more complicated. We have taken the images (virtual and real), formed at the far points in the two states of ametropia, for the sake of uniformity and for ease of demonstration and comparison in the two conditions.

§ 116. It is apparent from the foregoing, that it cannot be strictly true, as has been commonly stated in a general way without qualification, that the inverted image of the myopic eye is smaller than that of the hypermetropic eye when both are formed by the same lens. The relative size of the inverted image, as we have seen, depends entirely on the place occupied by the auxiliary lens on the optical axis, and there are some positions of the lens in myopia—when it is near the far point of the eye under examination—where it will be, not only relatively, but actually larger than the image of a hypermetropic eye of the same degree produced by the same lens in the same position as regards the eye.

§ 117. These general rules in regard to the size of the inverted image of the myopic and hypermetropic eye are equally applicable to the *myopic* and *hypermetropic meridians* of the *astigmatic* eye. The effect upon the image caused by the displacement of the auxiliary lens is of a peculiar character and different in the different forms of the anomaly. An observa-

tion of these changes is most useful in making the diagnosis of
the form of astigmatism from which the eye may suffer, and
gives us a general idea of the direction of the principal me-
ridians and, in a rough way, of the amount of the astigmatism.

§ 118. In the *simple* form of *myopic astigmatism*, when the
lens is gradually removed from its ordinary position at from 1
to 2 inches in front of the cornea towards the far point of the
eye, there will be a progressive enlargement of the optic disk
in the myopic meridian, while the diameter of the disk which
corresponds to the emmetropic meridian will remain un-
changed. If, for instance, the vertical is the myopic meridian,
the surface of the disk instead of being round will appear
drawn out vertically, and the amount and the rapidity of the
elongation will be in proportion to the degree of the myopia in
that meridian. Should the myopic meridian be oblique, the
inclination of the oval disk will correspond to the direction of
the faulty meridian. It must be borne in mind, in this connec-
tion, that owing to the inward rotation of the eye necessary to
bring the optic disk into view in ophthalmoscopic examina-
tions, the disk is seen somewhat in profile and in the normal
eye does not generally appear round, but vertically oval. For
the diagnosis of a myopia in the vertical meridian, therefore,
the oval in this direction must *increase* as the lens is removed
from the eye.

When, on the other hand, the lens is brought very close to
the cornea, the horizontal (emmetropic) diameter remains un-
altered, while the vertical (myopic) diameter gradually dimin-
ishes in size, assuming finally, if sufficiently close, a horizontal
oval form.

§ 119. In *compound myopic astigmatism*, we have, in accord-
ance with the facts just stated, a general progressive enlarge-
ment of the disk on removal of the lens, with a greater enlarge-
ment and drawing out of that diameter of the disk correspond-
ing to the meridian most strongly myopic, while there is a
greater shortening of the same diameter when the lens is
brought very close to the cornea.

§ 120. We have, of course, an entirely different set of

changes in *hypermetropic astigmatism.* Example: As the lens is withdrawn from the eye, the vertical diameter remains essentially unchanged, while the horizontal diameter, corresponding to the hypermetropic meridian, becomes more and more contracted. Diagnosis: *Simple hypermetropic astigmatism, axis vertical.*

§ 121. In *compound hypermetropic astigmatism* there is, on withdrawal of the lens, a narrowing of the disk in all its diame-

Fig. 33.
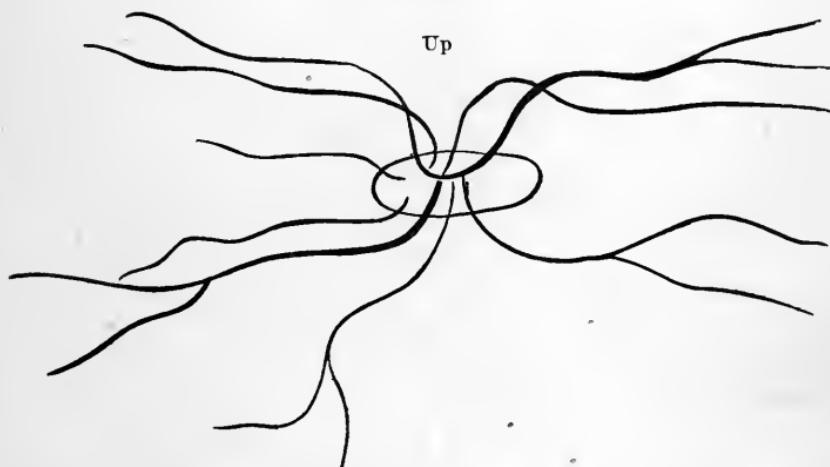
APPEARANCE OF THE FUNDUS OF AN EYE WITH MIXED ASTIGMATISM, WHEN THE AUXILIARY LENS IS HELD CLOSE TO THE CORNEA.

ters, but it is more rapid in the diameter corresponding to the most hypermetropic meridian. The result is an oval with its short diameter in the direction of the meridian of least refraction, the same as in the myopic forms, but it is produced in a diametrically opposite manner. In the hypermetropic forms it is the result of *contraction* in the faulty meridian; in the myopic forms it is due to *enlargement* in the direction of the faulty meridian.

§ 122. The most marked picture, however, is that furnished by the *mixed* form of astigmatism. Let there be, for example, (as in the case from which the accompanying drawings were taken), hypermetropia of 3 D in the vertical meridian, and myopia of 7 D in the horizontal meridian. We have here the con-

ditions necessary for producing the combined effects of both these optical states in a very pronounced manner. When the lens is held very close to the cornea the enlargement in the hypermetropic (horizontal) meridians is at its greatest, while the

Fig. 34.
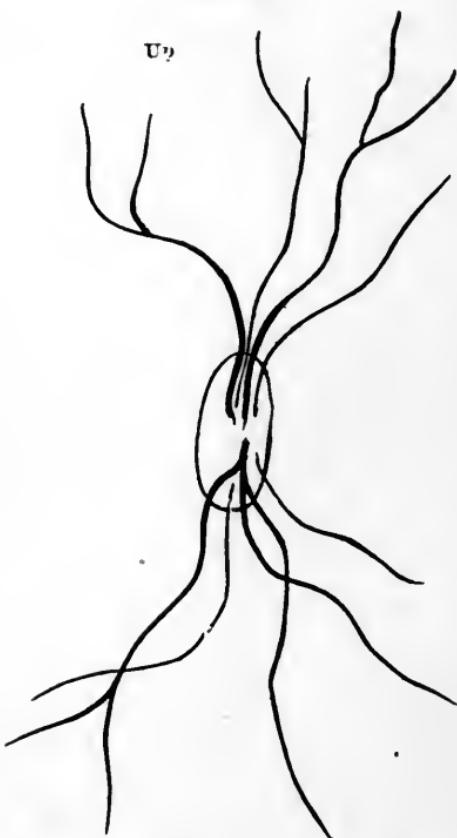
APPEARANCE OF THE FUNDUS OF AN EYE WITH MIXED ASTIGMATISM, WHEN THE AUXILLIARY LENS IS AT A GREAT DISTANCE FROM THE CORNEA.

contraction in the myopic (vertical) meridian is at its minimum. As a consequence the disk is elongated horizontally, while the retinal vessels are drawn together and take a more horizontal direction (Fig. 33). As the lens is gradually removed from this position the horizontal (hypermetropic) diameter progressively contracts, the vertical (myopic) diameter progressively enlarges, and the vessels turn, as it were, on a pivot, becoming

more and more vertical until a point is reached where we have the appearances given in Fig. 34 in which the enlargement in the vertical (myopic) meridian is greatest, and the diminution in the hypermetropic (horizontal) meridian approaches its minimum.

§ 123.   These examinations should be made with a lens of short focus—two inches at most—since a strong lens gives a large ophthalmoscopic field, which with a wide pupil, is almost indispensable for a satisfactory diagnosis.

§ 124.   The chief defect of the method is the lack of accuracy in determining the degree of astigmatism.   This, as well as the direction of the principal meridians, can be only roughly estimated by the rapidity of change in the form of the disk on moving the lens, and the direction of the long axis of the oval. Its value lies in the knowledge it gives us of the existence and form of astigmatism in those cases where the amblyopia or stupidity of the patient makes it difficult or impossible to obtain reliable data by any one of the subjective methods of examination.

§ 125.   Knapp and Schweiger have called attention to the change in the form of the disk, when seen successively by the direct and indirect methods of examination, as a means of diagnosing astigmatism.   The erect image of the disk in both M and H is oval, with its short diameter in the direction of the meridian of least refraction; in the inverted image, on the other hand, the short diameter coincides with the meridian of greatest refraction.   This, however, is true only when the auxiliary lens is held at the ordinary position near the eye. When it is removed to any considerable distance from it, as we have seen, there is a change to the opposite condition, and the oval is the same as in the direct method.

§ 126.   Bravais, of Lyon, suggested a method based on the associated movements of the lens and image.   When the disk is in the center of the lens and the latter is moved from side to side, if there is emmetropia the lens and image move together. In myopia the movement of the image is less than that of the lens; in hypermetropia it is greater.   By moving the lens in

various directions perpendicular to the optical axis, and noting the amount of displacement of the image, some idea may be formed of the direction of the principal meridians and the kind of refraction in each.

§ 127.   In this and in all the methods of examination with an auxiliary lens, great care should be taken to hold the lens strictly at right angles to the visual axis, for otherwise we would have the cylindrical action resulting from an oblique position of the spherical, which might be misleading as to the optical condition of the eye.

§ 128.   Mr. Couper has suggested a method of examination by the mirror alone for the observation of the inverted and erect images, successively, which is available particularly for the detection of the mixed and simple forms of astigmatism. For this purpose he employs a mirror of 30 inches focus and places himself at a distance of $4\frac{1}{2}$ to 5 feet from the patient.   He is then in a position to see an inverted aërial image of the fundus formed by the meridian having a myopia of 1D or more.   This image may be only a portion of a vessel or a part of the edge of the optic disk, and will be larger and formed farther from the eye the lower the degree of the myopia, while the direction in which the vessel or vessels appear to run will be at a right angles to the faulty meridian.   If the other meridian is emmetropic or hypermetropic, of course no vessels or other details of the fundus lying in that direction can have their images formed at that distance, and, therefore, cannot be seen.   These come into view only when the mirror is brought closer to the eye, when they will be seen directly and erect, while the vessels in the other meridian will have vanished.

§ 129.   Schmidt-Rimpler claims that his plan for estimating refraction by the inverted image is applicable for the diagnosis of astigmatism.   In this method of examination the observer does not look at the retinal vessels or other details of the fundus, but at the aerial image of a gas-flame or illuminated fret-work which serves as the source of illumination.

The manner in which this image is produced is as follows:

A fret-work of small squares and triangles, made in a screen, is placed before the flame of gas or a lamp. With a concave mirror of short focus (20 cm.) an image of this brilliant fret-work is formed in front of the observed eye, and becomes the source of its illumination. An auxiliary lens of a certain focus (10 cm.) is held at a fixed distance (10 cm.) from the eye to be examined, and the combined power of the lens and the re-fracting apparatus of the eye form an image of this open work in its fundus. This retinal image of the fret-work will be sharply defined only when the trellis-work image in the air and the fundus are at conjugate foci, and when this relation exists there will be formed, according to the law of conjugate foci, at the source of illumination a sharply defined image of the retinal image, and this it is which the observer sees.

Now, the distance from the lens at which this distinct aerial image will be formed depends, as we have seen in § 107 (the auxiliary lens and its distance from the eye being constant quantities), upon the refracting power of the eye. It is farther from the lens in H and closer to it in M than it is in E. If we have a means of measuring this distance, the determination of the kind and even the degree of ametropia becomes easy. This has been made possible by Schmidt-Rimpler, through a very simple arrangement. The power of the auxiliary lens and its distance from the eye being always the same, and the image of the fret-work being always at 20 cm. from the mirror, it is only necessary to approach the mirror to the eye until the aerial image becomes sharply defined, measure the distance between the mirror and lens, and subtract 20 from the dis-tance, in order to have the distance of the image from the lens. If a lens of 10 cm. focal distance is used, the image will be formed in emmetropia at 10 cm. in front of it, since the rays emerging from the eye strike it parallel; in myopia, the distance of the image from the lens will be shorter, in hyper-metropia, greater. For a lens of 10 D (10 cm. focus) it has been found that every variation of 1 cm. from this distance of 10 cm. corresponds to a difference in refraction amounting to 1D. This measurement he makes by means of a roller tape,

the roller end of which is attached below the lens, while the free end is held at the mirror.

The manner of making the observation is as follows: the auxiliary lens is fixed near the end of a small bar graduated in centimetres, and under it is the roller tape, which winds up with a spring. The free end of this bar is placed on the lower edge of the orbit, and when the lens is at 9.5 cm. it is about 10 cm. in front of the principal points of the eye. The mirror, to which the free end of the measuring tape is attached, is placed at 40 to 50 cm. from the auxiliary lens and the fundus of the eye illuminated in the usual manner. The aerial image of the fret-work will then be seen as an ill-defined bright spot. As the mirror is approached closer to the lens it becomes more and more clear in outline, and finally one position is found in which all the details are sharply defined. The distance of the mirror from the lens is then read off on the measuring tape and 20 taken from it. The remainder is the distance of the image from the lens. The difference between this number and 10 gives the amount of ametropia. If, for example, the distance between lens and mirror is 35 cm. the distance of the image from the lens is (35—20) 15 cm. and there is H of 15 — 10 = 5 D, each cm. corresponding to 1 D of refracting power. If it is 27 cm. the ametropia is M (27 — 20 = 7; 10 — 7) = 3 D.

In astigmatism, from what has already been abundantly demonstrated, it is apparent that in no single position of the mirror can the lines of the figures of the fret-work at right angles to each other be seen with equal clearness, since the image of one set of lines will be formed sharply at one place and that of the other at another. Taking advantage of this fact, we approach the mirror until the lines of the figure running in some one direction are seen with the greatest distinctness and, read off on the tape-measure the distance of the mirror. This gives us the refracting power of the meridian of least refraction. The mirror is then brought closer to the eye until the lines at right angles to these are seen sharply; the distance of the mirror read off on the tape then gives the re-

fraction in the most highly refracting meridian, and the difference between the two shows the amount of the astigmatism. Example: The lines running vertically are seen at 34 cm., those running horizontally at 31 cm.; there is compound hypermetropic astigmatism. In the vertical meridian there is H $(31 - 20 = 11; 11 - 10 = 1)$ of 1 D; in the horizontal H $(34 - 20 = 14; 14 - 10 = 4)$ of 4 D; H = 1 D with astig. = 3 D axis 90°.

§ 130. My own experience convinces me, however, and I believe the opinion will be corroborated by most ophthalmoscopists, that the two methods of examination by the erect and inverted images as usually employed, and as described in §§ 99 to 126, if cultivated with the care which their importance in other particulars demands, are capable of furnishing us with all the information in regard to the refraction of the eye that the ophthalmoscope is capable of giving.

The data thus obtained are often very accurate, but the liability to error is too great for any one of these methods to be relied upon implicitly. Before ordering glasses, the findings with the ophthalmoscope should be verified by an examination with cylindrical lenses and the test letters.

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CHAPTER VIII.

Skiascopy. (The Shadow-Test.)

§ 131. In a foot note on page 490 of his treatise on the 'Anomalies of Accommodation and Refraction of the Eye," Donders makes the following record:

" My friend Bowman recently informs me that ' he has been sometimes led to the discovery of regular astigmatism of the cornea, and the direction of the chief meridians, by using the mirror of the ophthalmoscope much in the same way as for slight degrees of conical cornea. The observation is more easy if the optic disk is in the line of sight and the pupil large. The mirror is to be held at two feet distance and its inclination rapidly varied so as to throw the light on the eye at small angles to the perpendicular and from opposite sides in succession, in successive meridians. The area of the pupil then exhibits a somewhat linear shadow in some meridians rather than in others.' "

This description is too short and indefinite to enable us to decide whether these experiments of Bowman held the germ of the method now known under the names of "keratoscopy," "retinoscopy," "pupilloscopy," "phantoscopy," etc., but at all events no extensive practical application was made of the changes observed in the pupillary area when illuminated from a distance by a simple mirror until the publication of an article on the subject by Cuignet in the Recueil d' Ophthalmologie in 1873.

§ 132. Most of the names given to the method do not convey any intelligent idea of the nature of the phenomena on which it is based. The appearances do not pertain either to the cornea, pupil or retina alone. They are due entirely to the refractive condition of the eye as a whole. As the principal and distinguishing feature of the method is the behavior of the

shadowy edges of the bright image of the source of illumination, "the shadow-test" would seem to be the most appropriate term by which it could be designated. If, however, we should wish to use a general scientific term, the word "skiascopy," as suggested by the celebrated Greek scholar, M. Egger, is available.

§ 133.   The method is founded on the observed fact that when the light from a flame, placed in the ordinary position for ophthalmoscopic examination, is thrown into the eye by means of a mirror at a distance of from 3 to 5 feet from the eye, and the mirror is rotated about one of its axes, a shadow is observed to pass across the bright area of the pupil. The relative brightness of the pupil and the direction and rapidity with which the shadow moves serve as a basis for diagnosis, and on the following principles:
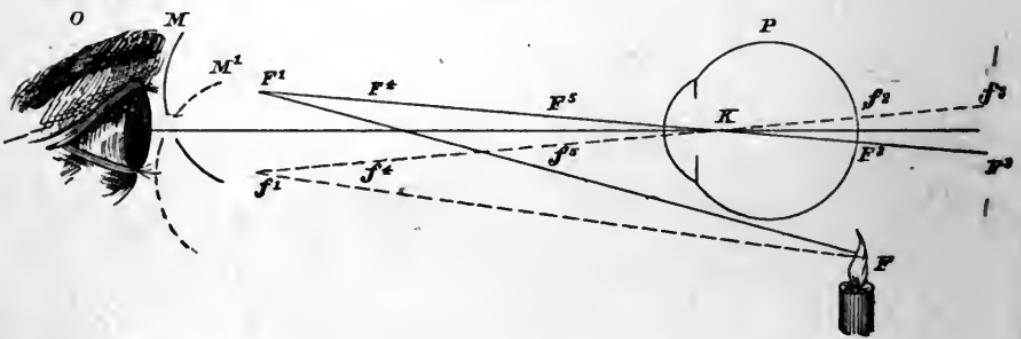
Fig. 35.

SHOWING THE OPTICAL PRINCIPLES ON WHICH SKIASCOPY IS BASED.

§ 134.   When the light from the flame F, Fig. 35, is thrown by a concave mirror M in the direction of the eye P, from a distance of from 3 to 4 feet, an image of the flame is formed, near the focus of mirror, at F^1.   This image then becomes the illuminated object from which rays diverge and which, passing into the eye P, form an image on its retina at F^2. The light is reflected from this retinal image and passing out of the eye forms in its turn an image real or virtual at a certain place before or behind the eye, according to its refractive condition.

§ 135.   Let us suppose, for illustration, that F^1 is one

meter from K the nodal point of the eye P.   Then, if P is em-
metropic, the image of F^1 formed on the retina at F^2, which in
this case will be blurred in outline, will send out rays that will
be rendered parallel by the refracting media and an image of
F^2 will be formed at infinity, behind the observer's eye at O.
When the mirror is rotated to the position of M^1, the source of
illumination moves from F^1 to f^1 and the retinal image moves
from F^2 to f^2.   To the observer's eye at O the movement of
the image of f^2 formed at infinity by the refracting media of P
is in the sense *contrary* to the movement of the mirror, and the
extent of this movement is *infinitely* large as compared to that
of the mirror.

If, however, the eye is myopic with its far point at F^4, the
retinal image F^2 will form a real and inverted image of itself at
that point.   When the mirror is rotated to M^1 the image of f^2
will move from F^4 to f^4 in the *same* sense with the mirror.
The *amount* of movement will be measured by the distance
from F^4 to f^4.   Should the far point of the myopic eye lie at F^5
the image of F^2 will be found there, and on rotation of the mir-
ror, move to f^5, likewise *with* the mirror.   The amount of move-
ment is measured by the distance, $F^5 f^5$.

It will be seen from this that the movement is less the greater
the degree of myopia, and greater the smaller the degree.

§ 136.   When, as in hypermetropia, the image of F^2 is *virtual*
and *behind* P, the character of its movements is changed.   If it
is found at F^3, for example, on rotation of the mirror to M^1 F^3
will move in an opposite direction to f^3 and the *extent* of move-
ment will be expressed, as before, by the distance $F^3 f^3$.   The
farther F^3 is removed from K, that is the lower the degree of
hypermetropia, the greater the amount of movement, until it
reaches its maximum when the rays coming from it become
practically parallel, and the eye essentially emmetropic.   The
closer F^2 approaches to K, that is the higher the degree of hy-
permetropia, the less the amount of movement.

§ 137.   From the foregoing the following facts are estab-
lished: 1. That in *emmetropia* and *hypermetropia* the move-
ment of the image is against that of the mirror.   2. That

in *myopia* of degrees above 1D, that is when the far point of the eye is within 3 feet, the movements are *with* the mirror.   3. That in both forms of ametropia the *higher* the degree the *less* the *extent* of movement.

§ 138.   But this is not all. The *brightness* of the image changes with the degree of ametropia.   If F^1 coincides with the far point of the observed eye (there being M of 1D), the image F^2 will be clear and distinct, and all rays proceeding from it will come back and be again united at F^1.   There will then be a maximum of brightness of the image seen by the observer O. If, however, F^2 should not be the conjugate focus of F^1 there will be circles of diffusion at F^2 which will be large in proportion to the departure in either way from this M of 1D.   Each individual point of F^2 will, therefore, have fewer rays coming from it, and there will be a diffusion of them over a greater extent of surface, with a corresponding diminution in distinctness of the image.

§139.   Now, what we specially note in this method of examination is the *shadowy outline* of this circle of diffusion which is so large in ametropia that the edges appear as almost straight lines, and its diagnostic value is summarized as follows :

1. Movement of shadow edge *against* the mirror, low degree of M, (less than 1.D) emmetropia, or hypermetropia ; *extent* of movement *less* the higher the degree of hypermetropia. *Bright reflex* ; E. or low degrees of M. or H : *dull reflex*, higher degrees of H.

2. Movement of shadow *with* the mirror ; M. above 1 D, its *extent* diminishing as the degree increases.   *Reflex* duller in direct proportion to the degree of M.

§ 140.   There is one objection to this examination when made with the concave mirror, namely, that by it we are not able to distinguish between emmetropia and low degrees of hypermetropia and myopia, since in them all the shadow moves *against* the mirror.   This is due to the fact that the source of illumination, F^1, is always at a finite distance and gives out divergent rays, and if it is situated at a greater distance from P than one meter, only a limited number of its rays will enter the eye, and

the illumination of the retinal image will be greatly diminished. This nearness of F^1 to P necessitates a near approach of O to P. As a consequence of this, in all cases of M, where the far point lies behind O, no actual inverted image of F^2 is formed in front of O, but on the contrary, it tends to become virtual and erect and is classed with E. and H., and has the same movement against the mirror.

In order to obviate this and have sharply defined diagnostic movements of the shadow, Story and others have suggested the use of a *plane* mirror, which would enable the observer to place himself at a distance of 12 feet or more from P. Under these circumstances the rays coming from the flame two feet behind P and fourteen feet from the mirror would approach parallelism and would be so reflected by the plane mirror into the eye P, without loss by dispersion. On emerging, the rays from F^2 could form a real image six feet from P which would be recognized by O and a M. of o 5 D distinguished. When the plane mirror is used, however, it must be borne in mind that the movements of the shadow are the *reverse* of those observed when the concave mirror is used; that is, *in H and E, it moves with* the mirror, *in M against it.*

The reason for this is as follows : When the concave mirror is used, the source of illumination is an image of the flame at the focus of the mirror, and when the mirror rotates from right to left this image also moves from right to left, and consequently the retinal image F^2 moves in the contrary direction, with the results as we have seen. With the plane mirror, however, the object furnishing the light is a virtual erect image of F situated as far *behind* M as the flame F is in front of it. When the mirror is rotated, therefore, from right to left, this image moves in a *contrary* direction and its retinal image, F^2, in the *same* direction as the mirror. The image furnished by F^2 must move, in accordance with this, in a direction relative to the mirror exactly the opposite to that followed in the examination with the concave mirror.

§ 141. The examination so far, however, has furnished us with no definite information as to the amount of ametropia.

We can know the higher degrees of M. or H., and if the plane mirror is used even the lower degrees, but we can only estimate the amount of departure from the normal optical condition in a rough manner by the greater or less dulness of the pupillary reflex, and the greater or less extent of shadow movement.

It is possible, however, to obtain a quite accurate diagnosis by this method, by placing in front of the eye under examination correcting glasses in succession until one is found with which emmetropic movements of the shadow are found.

Let there be, for example, a short movement *with* the concave mirror and a dull reflex. This indicates, of course, M., and experience teaches us, of a moderate degree. We then put a —4 D in a trial frame, and placing it before the eye, make another observation and note the direction of the shadow. It is found to still be *with* the mirror : the M. is under-corrected. After trying —4.5 with the same result we find that with —5 there is large movement *against* the mirror, and a maximum brightness of reflex. This shows that there is a small amount of M. (about 0.5 D) remaining uncorrected. Adding this to the 5 D we have 5.5 D as about the degree of M. present. Should the plane mirror be used, we find before correction a movement against the mirror, and not until —5.5 is placed in the frame do we find a movement with the mirror.

When one becomes skilful in the use of this method an approximation to within 1 D of the optical state of the eye can be made in the majority of cases. For its most satisfactory employment a wide pupil is essential, and it is necessary that the eye examined shall be protected thoroughly from all light except that which comes from the mirror.

Like most of the other methods of objective examination, it requires much practice to become expert in its use, and is, according to my experience, more consumptive of time than the ordinary ophthalmoscopic methods.

§ 142. It will be seen at once how easy it is to use this method in the diagnosis of astigmatism. We have, for this purpose, simply to determine, in accordance with the principles laid

down, the ametropia of the principal meridians in the same way that we examine for the general ametropia.   Moreover, the direction of the principal meridians is, owing to some peculiarities of the phenomena, very readily discovered. If these meridians stand as they commonly do, vertically and horizontally, there will be a difference in the amount or character of shadow movement on rotation of the mirror in these directions respectively.   But should the meridians lie obliquely—one being, say, at 45°—on horizontal movement of the mirror, the

Fig. 36.
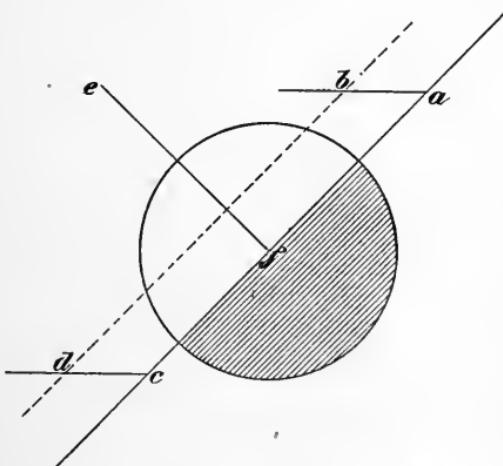
DIAGRAM SHOWING HOW THE SHADOW IN SKIASCOPY MOVES ACROSS THE PUPIL
WHEN THE MERIDIANS IN ASTIGMATISM ARE OBLIQUE.

shadow does not move in a horizontal direction, but at an angle of 45°, and for the following reasons:

We know, from the facts demonstrated in Chapter II, that both the diffuse retinal image of F^1 and the image of this image as seen by the observer, O, being formed by an astigmatic system are not circular as in general ametropia, but oval, with the long diameter in the direction of one of the principal meridians. The oval lying, as it does in our supposed case, with its long axis at 45°, advances, when the image as a whole moves horizontally, its shadowy edge in a direction *at right angles* to its axis.   Without going into any mathematical demonstration of

the matter it is easy to convince oneself of this fact by a very simple experiment. Take a circular opening (Fig. 36) and place behind it an object with a straight edge *a c* at an angle of 45°. Now advance this object in a strictly horizontal direction to *b d;* the *apparent* movement of the object will be, not from *a* to *b* and *c* to *d*, but in the direction of the line *e f* perpendicular to *a c*.

§ 143. In examining for astigmatism by means of the "shadow test" we first throw the light into the eye in the usual manner, and rotating the mirror in various directions note the brightness of the reflex, and the direction and extent of the shadow movements. If astigmatism is present, it reveals itself at once by the incongruity of movement explained in the preceding paragraphs, and the direction of the principal meridians is thus indicated. We then proceed to test the refraction of each meridian separately by placing in the trial frame before the eye various correcting glasses and rotating the mirror in the direction of this meridian until one lens is found which gives emmetropic motions. The meridian at right angles to this is then taken and the refraction determined in the same manner. Knowing then the refraction in the two meridians it is easy, in the way already sufficiently dwelt upon, to find the amount of astigmatism. When the correcting glasses thus indicated are placed in the frames with the axes of the cylinders at the proper angle, movements of the mirror in all directions should give emmetropic motions to the shadows.

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CHAPTER IX.

Keratometry and Keratoscopy.

§ 144. The fresh impulse given to the study of astigmatism in recent times was by measurements of the cornea in its various meridians, first made systematically by Knapp. And since the cornea has been found to play the chief part in the anomaly, the most convenient and proper method of investigation would seem to be by some kind of keratometry. And so it would be but for the fact that, until very recently, no rapid and accurate method of keratometry has been at the command of the practitioner. The ophthalmometer of Helmholtz is cumbersome, expensive, difficult to handle and very consuming of time; all of which tend to exclude it from the consultation room.

§ 145. The desirability of some practical method of keratometry being recognized, several attempts have been made to realize it, but up to 1881 none had been presented which offered the merits of accuracy, facility of handling, simplicity of structure and comparative cheapness. In that year, Javal exhibited before the International Medical Congress, at London, an instrument constructed by himself in conjunction with Shiötz which fulfilled all these requirements, and in my estimation must be regarded as the most important, practical and exact means of diagnosis given to our science since the invention of the ophthalmoscope.

§ 146. As regards the designation of this method of examination we think the name "keratometry" more accurately descriptive than "ophthalmometry." The latter term could be applied to measurements of the eye in any or all of its parts, whereas, in the method under consideration, we simply measure the radius of curvature of the cornea at any desired locality. Neither would "keratoscopy" be correct, since this means a simple inspection of the cornea and does not necessarily imply any

measurements; a term quite appropriate, however, to the methods of Placido and DeWecker, to be described later.

§ 147. The instrument of Javal and Shiötz [1] is constructed on the well-known fact that the image of an object of a certain size at a fixed distance from a convex reflecting surface grows smaller as the radius of curvature of that surface becomes shorter.

There are several ways in which this principle can be applied in determining the curvature of any given convex surface.

(*a.*) A certain size of image may be taken as a standard and the distance between the object and reflecting surface varied until this standard size of the image is reached. It can then be determined, by calculation, what changes in radius of curvature correspond to a certain variations in distance ; or

(*b.*) the distance separating the object and reflecting surface may remain the same while the size of the object is varied till the standard size of image is obtained ; or

(*c.*) the distance and size of object being fixed the calculation may be based upon the varying size of the image.

§ 148. Of these plans the first and second are the most available, and of the two, Javal and Shiötz have chosen the second.

The essential part of the instrument for ordinary use is a Wollaston's bi-refringent prism of such a power that it shall give an exact doubling of an object 3 mm in diameter when it is at 27 cm from the prism; or, expressed in another way, when the object is 27 cm from the prism and the edges of the double images touch, the diameter of the object must be 3 mm. The object to be measured in this case is the corneal reflection of some suitable object placed in front of the eye.

The accessories of the instrument are the optical appliances for seeing this double image to the best advantage and from a convenient distance, and a means for measuring the varying size of the object which furnishes the corneal image.

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[1] Made by M. Laurent, rue de l'Odeon, Paris. The net price of the instrument is 350 francs. It can be obtained through J. W. Queen & Co., Philadelphia.

In the construction of the *objective* W (Fig. 37) the birefringent prism is placed between two convex lenses having each a focus of 27 cm. At the focus of the second lens, found between E and O, a fine spider's web is stretched across the tube. By

Fig. 37.
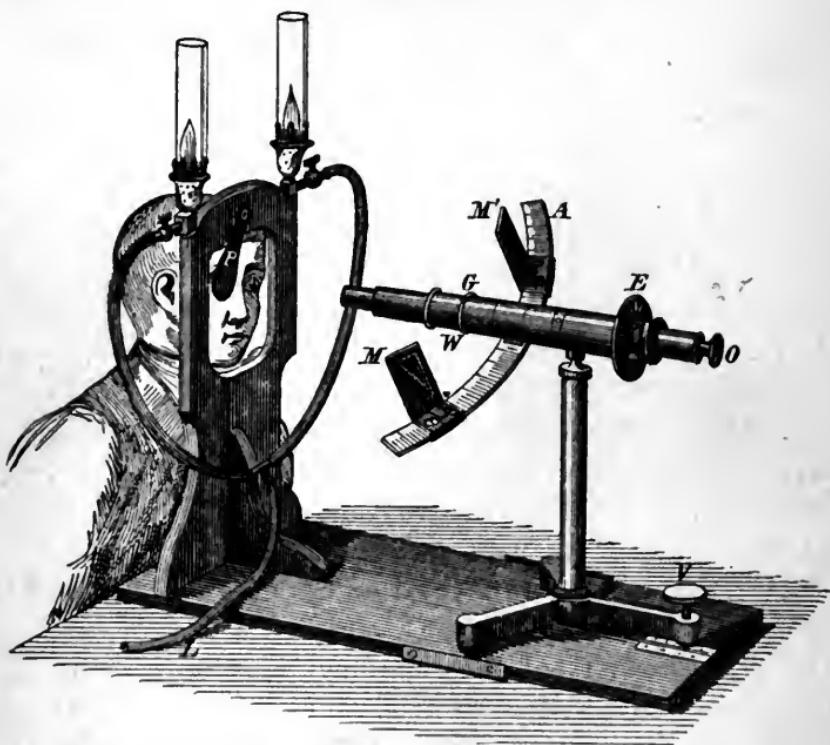
THE OPHTHALMOMETER (KERATOMETER) OF JAVAL AND SCHIOETZ.

means of these two lenses and the prism an inverted double image of the reflection from the cornea, which is situated 27 cm from the first lens, is formed at the spider's web, and this double image is seen through the *ocular* O whose focus also lies at the spider's web. This double image is seen clearly, only when the eye of the observer is accurately adjusted by means of the ocular to that distance. This inverted double image of the corneal reflection is not magnified. The reflection and its image are at conjugate foci, and the distance of the cornea from the first lens being the same as the distance of the spider's web

from the second lens, the two images must be of the same size ; in other words, the reflection image is simply transferred from the cornea to the focus of the ocular.

The only thing now needed to furnish the necessary means for an examination is an *object* whose size can be regulated at will. All that is essential of such an object are two opposing sides, in order that we may know when the edges of the double images are in contact. This is obtained by having two white bands $M M'$ moveable on an arc A, that is fixed on the tube W at about 35 cm. from the cornea which serves as its center of curvature. The outer edges of these two bands constitute the lateral limits of the object. As these bands are moveable it is easy to form an object of any desired size which can be accurately measured on the arc.

The instrument has now only to be properly mounted to be ready for use, and the manner in which this is done is well shown in Fig. 37.

§ 149. In making an examination the head of the examinee is placed in the head rest and the eye not to be examined is covered with the shade P. The optical part of the apparatus is moveable, as a whole, backwards, forwards and laterally on the foundation board, and the elevation of the tube is regulated by the thumbscrew V.

The ocular O must first be accurately adapted to the spider's web for the eye of each observer. The tube is then brought into line with the cornea and by movements back and forth so adjusted that a clearly defined double image of $M M'$ shall be formed at the spider's web, where it is seen by the observer through the ocular O. When this is done the prism is 27 cm. from the cornea, and when the double images of the corneal reflection formed by it touch by their opposing edges, each image has a diameter of 3 mm.

When the instrument is thus adjusted, one of the white bands is moved along the arc until the inner edge of its image, as seen through the ocular O, touches the outer edge of the image of the other band. Then, as stated above, the diameter of the corneal image of $M M'$ is 3 mm. The size of the object $M M'$

can then be measured off on the arc *A*.   Knowing now the
size of the object, the size of the image and the distance sep-
arating the object from the reflecting surface, we have all the
data necessary for calculating the radius of curvature of the
convex surface of the cornea.

§ 150. Thus far the instrument has no special practical ad-
vantage over the ophthalmometer of Helmholtz, for it is the
computations from the observed data that are so tedious and
time-consuming.   But Javal and Shiötz have so arranged the
various parts of their apparatus that a certain size of object
when the instrument is properly adjusted shall correspond to a
certain radius of curvature, which can be read off on the arc
in millimetres or expressed in refracting power by dioptries.
A variation of 6 mm. in the size of the object 35 cm from the
cornea corresponds to a change of 1 D in refracting power and
a variation of 36 mm. in size (6 D) corresponds to a difference
of about 1 mm. in the length of the radius of curvature.   As
it is comparatively easy by this instrument to approach to with-
in 0.25 D of the actual refracting power, an estimation of the
radius of curvature is possible up to $^1/_{20}$ mm., which is all that
could be demanded in the practical determination of refrac-
tion.

§ 151. As it is not possible with this instrument to examine
with any degree of accuracy the lenticular surfaces, and as
these form important elements in the general optical condition
of the eye, the apparatus is of limited advantage in obtaining
the refraction of the eye as a whole, except in cases of
aphakia.   Its usefulness for estimating general ametropia is
still further diminished by the fact, which the instrument itself
has done so much to establish, that the conditions of myopia
and hypermetropia are not due, except in rare instances, to
variations in the refracting surfaces of the eye, but to changes
in the antero-posterior diameter of the eye-ball.

§ 152. Since, however, it is a fact demonstrated more than
twenty years ago in the physiological laboratory, and now fully
corroborated by this very instrument in the consultation room,
that the much larger part of astigmatism is corneal, and since

it is as easy with it to measure the cornea in one meridian as in another, the value of this keratometer in the diagnosis of astigmatism can hardly be over-estimated.   In fact, it might with great propriety be called an "astigmometer."

The tube W can be turned on its axis, carrying with it the arc and the white bands $M M'$, and any departure from its initial position is shown by a pointer moving over the fixed graduated disk E.   It thus becomes possible to measure the corneal curvature in any desired meridian and to know the exact direction of that meridian.   We have, therefore, only to measure the cornea in the manner above described in its various meridians, read off on the arc the radius of each and take the difference between the strongest and weakest in order to have the amount of astigmatism expressed in difference in radius (r) or in dioptries.

The inventors have facilitated this determination in a most ingenious manner, whereby it is possible to see in the double image itself, and without the necessity of reading on the arc, the difference in the refraction of the principal meridians. One of the sides of the band M, instead of being straight is made in the form of five equal steps, as shown in A, Fig. 38.   It is the lateral edge of the lower step which must be brought in contact with the edge of the other band M' in order to have the image of the required size of 3mm.

§ 153. In making examinations with the instrument for astigmatism the tube is turned on its axis and the band M' moved on the arc, M remaining stationary, until one meridian is found in which the bases of the two bands form a continuous line, when their sides are in contact.   The refraction of the cornea in this meridian (indicated by the pointer on the disk E) is then read off on the arc in r or in dioptries and noted. The tube, with the arc, is then rotated to the opposing meridian, when it will be found, should astigmatism be present, that the sides have either separated or overlapped.   If they have *overlapped*, the *amount* of crossing is shown by the *number* and *portions* of the steps of M covered by M', which is readily recognized by the fact that these steps and parts of steps are just

twice as bright (for obvious reasons) as the remaining steps of
the band.   Now, the size of these steps has been so regulated
that each one shall represent 1 D of refracting power.   If,
therefore, one of these steps is covered, there is a difference of
1 D in the two meridians; if two are covered (B, Fig. 38) we

Fig. 38.
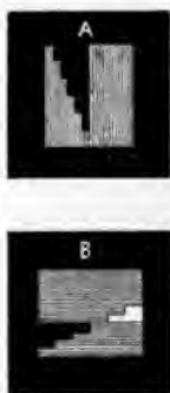
APPEARANCE OF THE CORNEAL IMAGE OF THE BANDS IN THE KERATOM-
ETER WHEN THERE IS A DIFFERENCE IN THE REFRACTION IN THE TWO MERIDIANS
OF THE CORNEA.—A, meridian of greatest curvature; B, the meridian of least curva-
ture (longest radius).

know the difference is 2 D and the necessity of reading the
difference on the arc is obviated.   If, however, it is desired to
know the radius of curvature exactly in this meridian, indica-
ted by the pointer on the disk E, the band M' is moved on the
arc until its edge is just in contact with the lower step of the
graded band M.   Its position on the arc will then give the de-
sired information.

Moreover, the meridian in which there is a crossing of the
bands is the less refracting.   The fact of the two images over-
lapping shows that they have a diameter greater than 3 mm.
and consequently the surface giving them must have less cur-
vature than that giving them with edges in contact, and in or-
der to have them thus in contact the object must be made
smaller by moving M' on the arc towards M.   If the images
of the bands *separate* in moving the arc from its initial position

where they were in contact, it shows that the first meridian is the less refracting with a larger radius of curvature. The curvature of the meridian where the separation is greatest is determined by moving the band M′ along the arc away from M until the edges again touch. The refraction can then either be read off on the arc and the difference taken between that and that of the other meridian, or the arc can be turned again to the opposing and less refracting meridian, when the number and parts of steps overlapping will show at once the difference between the two meridians, expressed in dioptries and fractions thereof.

§ 154. We have now ascertained the amount of the astigmatism and the direction of the meridians of greatest and least refraction, but the data furnish no clue as to the form of the anomaly. The measurements give no information as to whether the astigmatism is simple or compound, myopic or hypermetropic, or mixed. When, for instance, the horizontal meridian is found to be least refracting, it may be that (a) it is emmetropic, while the vertical meridian is myopic—simple M. astig.; or (b) it may be myopic also but less so than the vertical—comp. M. astig; or (c) it may be hypermetropic, the vertical being emmetropic—H. astig; or (d) it may be that both meridians are hypermetropic, this one being more so—Comp. H. astig.; or (e) it may be hypermetropic while the other is myopic—mixed astig. The differential diagnosis of the form must therefore be made by some of the other methods of examination.

The only other defect of the method is that it leaves us in ignorance of the amount of the lenticular astigmatism. One important fact has been developed by the use of the instrument in this connection and that is that in at least one half the cases it is neutralizing in its character. When the individual is beyond 40 years of age, however, it is rare to find any important difference between the total and the corneal astigmatism.

In the following table are recorded 15 cases taken as they came from my note-book showing the difference between the astigmatism as determined by glasses and that found on meas-

urement of the cornea. The excess of corneal refraction is des-
ignated by +, the deficiency by —.

TABLE VI.

No.	Name.	Age.	Astig. by Glasses.	Astig. by Keratometer.	Difference.
1	Mr. N.	48	L 1 R 1.5	1 1.5	
2	Mrs. H.	50	L 1.5 R 1	1.5 1.	
3	Mrs. C.	52	L 2 R 2	2.5 2.5	+0.5
4	Mrs. M.	21	L 4 R 1.5	4. 1.25	—0.25
5	Miss S.	28	L 1.25 R 3	1.25 3.25	+0.25
6	Miss C.	17	L 0.5	0.75	+0.25
7	Mrs. M.	38	R 1.25	1.25	
8	Mr. L.	19	R 0.5	nil	—0.5
9	Mr. G.	8	R 0	0.5	+0.5
10	Miss G.	18	R 0.5	0.5	
11	Miss C.	36	L 3 R 3	3.25 3.25	+0.25 +0.25
12	Mr. G.	50	L 4.5 R 4.5	4. 4.25	—0.5 —0.25
13	Miss L.	18	L none R 0.5	0.5 1.	+0.5 +0.5
14	Mrs. C.	33	L 3 R 4.5	3. 4.	—0.5
15	Mr. M.	27	L none R 0.25	0.5 1.25	+0.5 +1.

These cases well represent the average of my examinations
by this instrument which now number more than 100. I have

never found a greater difference than 1D, and the direction of the principal meridians has always been found to correspond essentially with the corneal meridians as determined by the keratometer.

§ 155. The apparatus, for its most satisfactory employment, requires a good illumination of the white bands. I find that in this climate the light from an unobstructed window is quite sufficient. But for dark days and consultation rooms not well lighted, the instrument is provided with argand gas-burners and reflectors which make it possible to use it under all circumstances.

Fig. 39.

THE KERATOSCOPE OF PLACIDO.

§ 156. PLACIDO'S KERATOSCOPE. In 1880 Placido, of Porto, described an instrument for noting changes in the form of the cornea by means of the reflection from its surface of a series of concentric circles. This is the simplest form of keratoscopy and is available to any one at the cost of very little trouble. Take a board of any inflexible material—wood, sheet-iron or stiff paper—25 or 30 cm. square, and draw on its white surface 5

concentric black circular bands 1 cm in width and $1\frac{1}{2}$ cm.
apart from each other (Fig. 39).   Make a hole in the center of
the disk through which to look, and you have all the essential
parts of the instrument.   The patient is to be placed with the
back to a good light and the board held at right angles to
the visual axis at from 8 to 12 inches from the eye under ob-
servation.   If there is no astigmatism, the reflex from the apex
of the cornea is circular and there is no irregularity in the
course of the circles.   Should regular astigmatism be present,
the corneal reflex will be oval with its long diameter in the di-
rection of the meridian of least refraction.   For the purpose of
enlarging the image, as well as for relieving the accommoda-
tion of the observer a convex lens can be placed behind the
hole in the disk.

§ 157. Javal has added one of these disks to his ophthalmo-
meter.   The principal value of this disk is, as we shall see, in
examining for irregular astigmatism.   It is of comparatively
little use in the determination of the regular forms.   At least
I, after considerable experimentation with it, have not been
able to satisfy myself of the existence of astig. below 3D. The
variation between the meridians of least and greatest refraction
is represented, as Javal's instrument shows, by a difference in
radius of curvature of about $\frac{1}{6}$ of a millimeter for one dioptre of
refracting power.   It requires a most expert eye, to detect in
the corneal reflection a difference of $\frac{1}{3}$ of a millimeter in the
radii of two opposing meridians, as expressed by the departure
of the figure from a strict circle. For a simple rough estimation
of the higher degrees it is, however, of some value, particularly
in the way Javal has adapted it to his instrument.

Fig. 40.
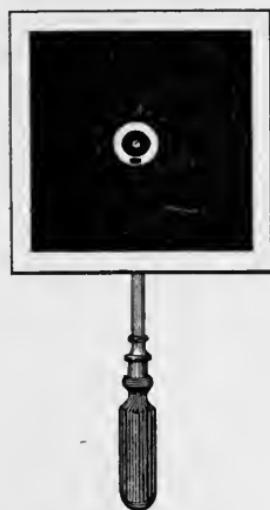
WECKER'S SQUARE FOR TESTING FOR CHANGES IN CORNEAL CURVATURE.

Fig. 41.

FIGURES FOR DETERMINING THE AMOUNT OF CHANGE IN THE CORNEAL REFLECTION OF WECKER'S SQUARE.

§ 158. The same may be said in a general way of De Wecker's modification of Placido's idea. This consists of a square instead of a circle (Fig. 40) whose corneal image is to be observed. He has made an addition to the method in furnishing a standard of comparison for the corneal image of the square by placing on a black surface (Fig. 41) the parallelograms which correspond to the different degrees of astigmatism expressed in dioptries. This is held near the eye under observation and a direct and simultaneous contrast of the two outlines is thus furnished, which enables the observer to judge the more readily as to whether there has been any departure from the strict form of the square and, if so, to estimate roughly how much.

§ 159. We have now completed a description of all the important methods for the diagnosis of astigmatism. Some, it has been seen, have an advantage in one particular and some in another, while all have certain insufficiencies. A preference for one over the other will generally be due to the fact that the individual has practiced it more assiduously than the rest and thereby attained greater skill in its use.

§ 160. *But no one method should be relied upon exclusively; and no diagnosis of astigmatism should be considered as fixed until it has been verified by an examination with cylindrical glasses and test-types as described in Chapter III. The highest visual acuteness must always be the final test of a diagnosis.*

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CHAPTER X.

Symptoms of Astigmatism.

§ 161. From what has gone before it will be readily understood that one of the most prominent, in fact a characteristic, subjective sign of astigmatism is diminished visual acuteness; and so it is when we come to examine these eyes in a scientific manner. But it is not always impaired vision which brings astigmatics as patients to the surgeon. A very considerable amount of bad vision often escapes the notice of ordinary patients suffering from astigmatism. Having never seen well at any time in their lives they have no standard of comparison in their own experience and a careful education of the sense of vision has made up largely for indistinct retinal images, and so it happens that they not infrequently flatter themselves that their visual acuteness is above the normal. "My eyesight is very good; I can often see things other people can't," is an expression heard almost every day in the consultation-room, and great is the astonishment of these patients when a rigid examination by the test-types shows a reduction of vision to $^1/_2$ or $^1/_3$ of the normal.

§ 162. Persons with degrees of astigmatism even above the medium may reach middle life without being conscious of their defect, and then their attention is only called to it when their presbyopia drives them to an examination for glasses. An instance of this is the following:

Mrs. G., aged 49, came to my office one day recently with her daughter who was under care for an asthenopia associated with hypermetropia, and mentioned casually in conversation that she had given up reading in the evening because she could get no glasses to suit her. She attributed this to age and had resigned herself to the situation. Upon my suggesting that probably some glasses might be found which would give her good vision she submitted to an examination. It was found that in either eye unaided by glasses V=$^4/_{24}$. In the left with +0.75 spherical it was somewhat improved, but with no other spherical glasses was it materially benefited. The

only line in Snellen's fan that she could see without glasses, as a line, was the horizontal, and this was also seen with $+0.75$. Other $+$ and $-$ glasses made it more indistinct. It was only when a $+5.25$ spherical was placed before the eye that the center line came out sharply, and then the other lines approaching the horizontal became totally indistinguishable. With a $+0.75 \subset +4.5^{cy}$ axis 90° vision came up to nearly $^4/_4$ and the fan was uniform. An examination of the R. E. made in the same manner showed an obliquity of the axis of the meridian approaching the horizontal of 10° to the outer side, and this was confirmed by the keratometer of Javal. This instrument showed, when the arc was at 100°, $R=8^1/_4$ mm.; when it was at 10°, $R=9$ mm. and in the latter position there was an overlapping of the double images of $4^1/_2$ steps on the graded band. With $+0.75 \subset +4.5$ axis 100°, $V=^4/_5$. The keratometer also confirmed the diagnosis of the astigmatism of the L. E., both as to degree and direction of the meridians. She had never suffered from asthenopia and during her life had done a great deal of fine needle work. A new world was opened up to her by means of her correcting glasses, she being then for the first time aware that she did not enjoy ordinarily good vision and could hardly be convinced that most people saw as well without glasses as she did with them. With $+2^s$ added to her correction for distance she was able to read the finest print in the evening for any length of time.

§ 163. Cases as extreme as the above are exceptional, but it is not at all uncommon for the lower forms of astigmatism, from 0.5 D to 1 D to make themselves felt first when the accommodation begins to fail. Young himself said, speaking of the influence of his astigmatism of $^1/_{23}$ on the acuteness of vision that "he believed he could examine minute objects with as much accuracy as most of those whose eyes are differently formed." Javal is of the opinion that in childhood and youth there is very commonly a masking of the corneal astigmatism by means of a partial accommodation, believing that when lenticular astigmatism is present, it is, in the majority of cases, neutralizing in its character. When age begins to affect the plasticity of the lens and the activity of the ciliary muscle, this power of unequal accommodation is lost and the latent astigmatism becomes manifest. How far this is true will be shown by a more extensive use of the keratometer, and a comparison of the corneal with the total astigmatism before and after paralysis of the accommodation.

§ 164. But a complaint of astigmatics more common than dimness of sight, though generally associated with it, is *asthenopia*, or painful vision. When the eyes are used for close

work, such as reading, writing, fine sewing, etc., for any considerable length of time, there is pain accompanied frequently with a sudden indistinctness of vision when the effort is prolonged. This asthenopia may be of all degrees of intensity from a slight feeling of fatigue to a pain so severe as to practically make any use of the eyes for near work impossible.

The asthenopia of astigmatism is of two kinds, which are usually denominated *muscular* and *nervous*.

The first named form has its seat in the muscle of accommodation, being sometimes called accommodative asthenopia, and the fatigue comes from the irregular and spasmodic contractions of the ciliary muscle. The eye instinctively endeavors to have as clear and distinct retinal images as is possible. An image distinct in all its parts is impossible in astigmatism, but by a kind of "see-saw" action of the ciliary muscles, first one part of the object and then the other can in the majority of cases have its image properly focussed on the retina. So long as it possible to see more clearly and satisfactorily by this kind of muscular action there is an irresistible temptation to use it; and, as is well known, nothing is more wearing on the muscular energy. A regular systematic contraction of a muscle may be continued for an almost indefinite time without fatigue, but convulsive-like movements soon exhaust its power. When general hypermetropia is present, and sometimes when it is not, this fatigue of the muscle is often followed by a suspension of its action with consequent indistinct retinal images.

In the highest degrees of astigmatism where no amount of accommodation can give distinct retinal images of any portion of objects there is no temptation to strain it, and as a consequence we do not find asthenopia of this kind so often a symptom in the higher degrees as in the lower.

§ 165. As is the case in the other forms of accommodative asthenopia, the pain is not usually referred to the eyes themselves, but manifests itself under some form of *headache*. The pain may be localized in any or all of the branches of the fifth pair of nerves, but in a majority of cases the headache is fron-

tal with a sense of constriction across the brow. It may, however, be general, and in some instances purely occipital, and its connection with the eyes remain for a long time unsuspected. The irritation may be reflected to other parts of the body and manifest itself under many curious forms, nausea being a not uncommon one.

The relation of headaches and anomalies of refraction as effect and cause has been long known to ophthalmologists, but the profession at large in this country were not impressed with its importance from the point of view of general medicine until Dr. S. Weir Mitchell called their attention to it in 1876. The general practitioner to whom these patients first appeal for the relief of their persistent headaches is very liable to overlook their real cause and will, of course, fail to give the desired relief, as the following case well shows:

Mrs. P., aged 40, had been affected with severe headaches for many years and suffered much at the hands of many physicians for its relief. The attacks would occasionally come on without any assignable cause, but would always follow any attempt to use the eyes. Her indistinctness of vision was not such as to call the attention of her medical attendants to her eyes, and the pain in them was considered as a part of the "neuralgia." It was impossible for her to read for more than ten minutes without bringing on an attack. Finally, owing to the death of her husband, it became necessary for her to enter as clerk in one of the departments where she would be compelled to use her eyes for eight hours a day. This, under the then existing circumstances, was impossible. In her despair she applied to still another physician who, suspecting that the refractive condition of the eye might have something to do with the trouble, sent her to me for examination. I found a simple hypermetropic astigmatism of $1/14$ axis of the correcting cylinders, R 45°, L 135°. With these $V = {}^{x0}/_{30}$, and she was able to read with them with proper correction of presbyopia without any considerable inconvenience or pain. She obtained her place in the Department and went immediately to work, and for more than six years has continued to use her eyes at very trying work from 9 a. m. to 4 p. m. without any material discomfort, and her headaches have entirely ceased. Occasionally when run down by work in the hot weather her eyes trouble her somewhat, but a few days rest brings her back to a condition of ordinary comfort. There is no ophthalmic surgeon of experience but can produce cases as severe in their character followed by relief as speedy as this one from the proper adaptation of glasses.

§ 166. The other form of asthenopia is *nervous*. The term is a broad one, but it must necessarily be so in order to cover the vagueness of our knowledge concerning it. The pain is

not muscular in its character and is not always dependent upon close application of the eyes in near work. Moreover, it is most commonly found in persons of neurasthenic tendencies. The fatigue is a mental one if we may so express it. Indistinct images are abhorent to the visual consciousness in the same manner as discordant sounds are abhorrent to the auditory consciousness, and there is an instinctive tendency to get away from them. Something of this feeling may be experienced by the emmetrope on rendering himself artificially astigmatic by means of a pair of cylinders.

The manifestations of this asthenopia are frequently feelings of general discomfort, associated, it may be, with dizziness, nausea and even vomiting. But often it is one of actual pain referred not uncommonly to parts not connected directly with the eyes. As in other forms of neurasthenic asthenopia, there may be intolerance of artificial or glaring light, and all kinds of abnormal sensations referable to the eyes and their adnexa. Chorea and other nervous disturbances of a general nature are laid at the door of astigmatism.

§ 167. These symptoms as well as those of muscular asthenopia sometimes make their appearance suddenly after a severe illness or as the result of some depressing causes operating to lower the tone of the nervous system, and may be the first intimation to the patient of the existence of an astigmatism. They continue in a greater or less degree in some cases, after the refractive anomaly has been corrected, necessitating most careful and systematic use of the eyes.

§ 168. Both forms of painful vision as well as the complaints of diminished visual acuteness are more common when the meridians are oblique than when they are horizontal and vertical. The reason of this most probably is, that when the meridians lie near the horizontal and vertical, it is possible by using one or the other focal plane to see clearly at least a part of the letters whose strokes run usually in these directions; when the meridians are oblique the lines forming the majority of the letters are blurred.

§ 169. Objectively there is little to be seen different from the

normal. There is, however, sometimes found a persistent frown associated with a narrowing of the palpebral aperture. This is the result of the habit the astigmatic has acquired of cutting off some of the circles of diffusion by means of the lids, thereby diminishing the indistinctness of the retinal images.

§ 170. Persistent blepharitis and chronic hyperæmia of the conjunctiva are frequent accompaniments of astigmatism as well as of the other refractive anomalies, and when found on examination of a patient should always lead to an investigation of the optical condition of the eye. It often happens that these conditions when associated with errors in refraction disappear as if by magic when correcting glasses are worn.

Dr. Martin of Marseilles ascribes a form of keratitis to astigmatism, but it seems to us that sufficient data are not at hand to establish a positive connection between the two as cause and effect.

§ 171. It is quite common for astigmatics, even those having the hypermetropic form, to consider themselves myopic, because they have to bring fine objects close to the eye in order to see them distinctly. The true explanation of this nearsightedness is that on a near approach of the object the increase in the size of the retinal image makes up to some extent for its indistinctness of outline.

§ 172. Astigmatism has also been considered as an active factor in the production of true myopia, and quite a number of cases have been cited in which simple myopic or hypermetropic or comp. hypermetropic astigmatism have been observed to pass over into the myopic forms. As the same can be said of general hypermetropia, it would seem that astigmatism could hardly be charged with this *per se*. It is quite possible, however, that the close approximation of the work mentioned in the preceding paragraph as the result of bad vision might ultimately lead, with an existing predisposition to myopia, to the development of a true axial myopia, and it is also possible that under these circumstances some of the asthenopia complained of is due to the unusual strain on the internal recti muscles.

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CHAPTER XI.

CAUSES OF ASTIGMATISM—LENTICULAR ASTIGMATISM.

173. Astigmatism may be congenital or acquired. With rare exceptions regular corneal astigmatism is *congenital* and in quite a considerable percentage of cases hereditary. It is by no means uncommon to find several members of the same family affected with astigmatism, though it may be of different kinds and in various degrees.

§ 174. As all eyes are more or less astigmatic, and abnormal differs from normal astigmatism only in degree, it is a rational inference that the manner in which the eye is developed has something to do with this peculiar formation of the cornea. Moreover, we should expect its shape to be influenced, to some extent, by the manner in which the tissues surrounding it— lids, orbit, etc.—are developed. Dealing now in what we acknowledge to be pure speculation, it seems probable that the formative influences at work on the eyeball, would result, if left alone, in the production of a spherical form of the cornea; but from some cause, inequality in pressure, unequal traction or the like, the sphere is compressed and the result is a spheroid such as we find it.

Javal, among others, has attempted to trace a connection between the general shape of the skull and the shape of the eyeball as expressed in its astigmatism. In fact he has formulated a law which he thinks warranted by the facts in the case, which is: "The meridian of greatest refraction corresponds to the shortest diameter of the skull." This law has not, we believe, been fully verified by the observation of others, though there is unquestionably something of truth in his general proposition. Further observations are needed on this point which has also a high ethnographic interest.

§ 175. Regular astigmatism may also be *acquired*. Any form of traumatism affecting the anterior portion of the eye-ball may have, as a result of its cicatrization, an altered curve of the cornea. The most common of these traumas is the *extraction of cataract*. That so extensive an incision as that employed in the various forms of extraction, should leave behind it some change in the corneal curvature is not at all astonishing. It can only be exceptionally that the coaptation of the wound would be so perfect as to leave no trace on the form of the corneal surface. These suppositions have been fully confirmed by measurements of the cornea before and after the operation. When the cornea is measured within ten or fourteen days after the operation, the meridian of least curvature is found, with rare exceptions, to be at right angles to the direction of the incision, and as the section is generally made upward or downward, the less refracting meridian is the vertical.

The amount of astigmatism at this period is sometimes enormous. In one of my cases, where I had reason to suppose a tardy union of the wound, it amounted to 15 D., and Laquer reports one of 16 D. minus refraction. Such great differences should always lead us to suspect some interference with prompt healing. When the progress of the case is normal, the astigmatism commonly begins from the tenth day to diminish, and occasionally a complete cicatrization may bring about a shortening of the radius in this meridian, rendering it the most highly refracting. In one case under my observation, the refraction in the vertical meridian passed over from 11 D. fourteen days after the operation to 20 D. eight weeks after. Further observations and examinations may lead us to the discovery of that form of incision and its position which is less likely to leave an important permanent deformity of the corneal curve. All kinds of *perforating wounds* and *ulcerations* of the cornea, particularly at its margin, very frequently leave behind them, in addition to the usual irregular astigmatism, a greater or less amount of the regular form which can often be corrected with decided benefit to vision.

§ 176. *Keratoconus, keratectasia* and other similar alterations

in the general form of the cornea, while giving in the majority of cases a very irregular refraction may yet oftentimes have an associated regular astigmatism which can be corrected with most decided benefit to the patient, as the following case shows:

Mrs. R., 34 years old, saw well up to her sixteenth year. V then began to fail and at her twenty-first year was at its worst. Since that time it has remained stationary. At the present time V is less than $4/60$. An examination showed the existence of keratoconus (in which aspect it will be considered under that heading), but measurements with Javal's keratometer also revealed a regular astigmatism. In L at $5°$ to the outer side of the apex, $180°$ R$=5$ mm. (39.5 D), $90°$, R$=6$ mm. (34 D), with a crossing of the bands of 6 steps; in R, at the same place, $180°$ R$=6^3/4$ mm. (30 D); $90°$ R$=5^1/2$ mm. (36 D), with a crossing of the bands to the amount of $6^1/$ steps. With $+4$ $180°$ $\bigcirc -3, 90°$ in R, V$==4/24$; with -7, $90°$ in L V$=4/12$.

In the investigation of such cases the ophthalmometer of Javal is of the highest value, and many cases which have heretofore been confined to the stenopaic slit or subjected to operation will find a measure of relief at least from properly adapted cylinders.

§177. *Lenticular Astigmatism.* The first case of regular astigmatism of which we have an accurate history was lenticular in its character. As a matter of historic interest, I give the exact words in which Young describes his condition as found in *Trans. Phil. Soc. of Lond. 1801* (not 1793 as stated by some), pp. 39-40, and contained in his paper on the "Mechanism of the eye."

" My eye, in a state of relaxation, collects rays which diverge vertically from an object at a distance of 10 inches from the cornea, and the rays which diverge horizontally from an object at 7 inches distance. For if I hold the plane of the optometer (fine wires) vertically, the images of the line appear to cross at 10 inches; if horizontally at 7. The difference is expressed by a focal length of 23 inches. I never experienced any inconvenience from this imperfection, nor did I ever discover it until I made these experiments; and I believe I can examine minute objects with as much accuracy as most of those whose eyes are differently formed. On mentioning it to Mr. Cary he informed me that he had frequently taken notice of a similar circumstance, and that many persons were obliged to hold concave glasses obliquely in order to see with distinctness, counterbalancing by the inclination of the glass the too great refractive power of the eye in the direction of that inclination (cor. 10. Prop. IV), and finding but little assistance from spectacles of the same focal length.

The difference is not in the cornea, for it exists when the effect of the cornea is removed by a method to be described hereafter. The cause is without doubt the obliquity of the uvea and of the crystalline lens which is nearly parallel with it, with respect to the visual axis ; this obliquity will appear from the dimensions already given to be about 10°. Without entering into a very accurate calculation the difference observed is found (by the same corollary) to require an inclination of 13° and the remaining 3° may be easily added to the greater obliquity of the posterior surface of the crystalline opposite the pupil. There would be no difficulty in fixing the glasses of spectacles or the concave eye-glass of a telescope in such a position as to remedy the defect."

Young, eliminating the refraction of his cornea by immersing it under water, demonstrated that the astigmatism of his own eyes was resident in the lens. This was its commonly accepted seat until the ophthalmometric measurements of Knapp suggested that the cornea might also play a part. As already stated in Chapter III it is now a well demonstrated fact that the cornea is the principal seat of the anomaly. The lens, however, not unfrequently adds its quota to the making up of the total astigmatism of the eye ; sometimes by increasing and sometimes by diminishing that of the cornea.

§ 178. The lens may effect an astigmatic action in two ways; (a.) by a malposition and (b.) by a change in the refraction of its meridians.

A dislocation of the lens in order to produce an astigmatic effect must so change its position on the optical axis that it shall lie obliquely to the direction of the rays striking its surface, as considered in § 25 ; a simple displacement perpendicular to the optical axis will not give rise to astigmatism. Young, as already stated, attributed his astigmatism to such an oblique position of the lens.

Regular astigmatism has also been found after iritis and after an operation for pterygium. Also in cases of pyramidal cataract and in membrana pupillæ perseverans.

Some of the cases of astigmatism following blows on the eye are in all probability due to such a dislocation of the lens. The *form* of the lens is so subject to the action of the ciliary muscle that it is only in exceptional instances that we can consider the lens curvature apart from the influence, active or
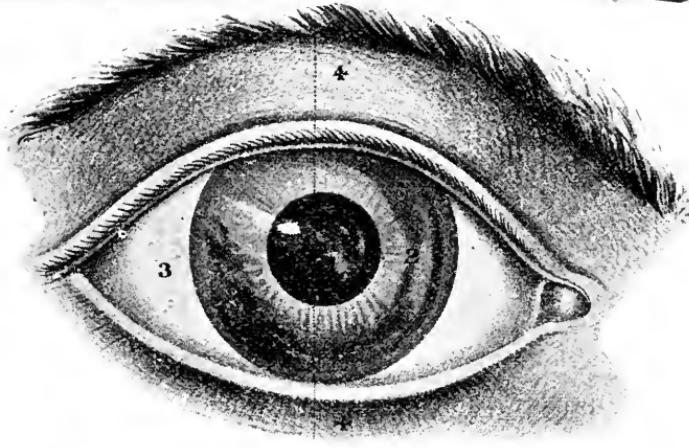
FIG. B. The Eye Ball Showing the Coats &c. of the Eye.
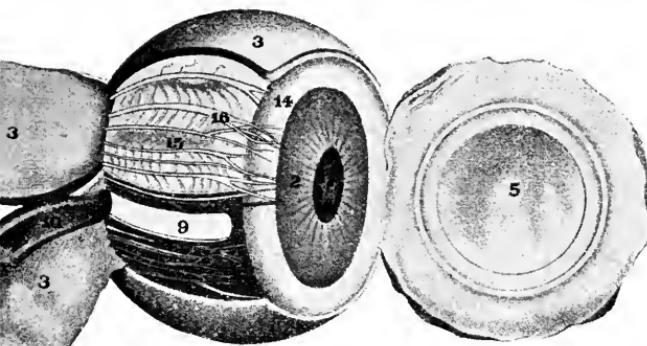
FIG. C. Longitudinal Section of the Eye and Orbit through the Dotted Line on FIG. A.

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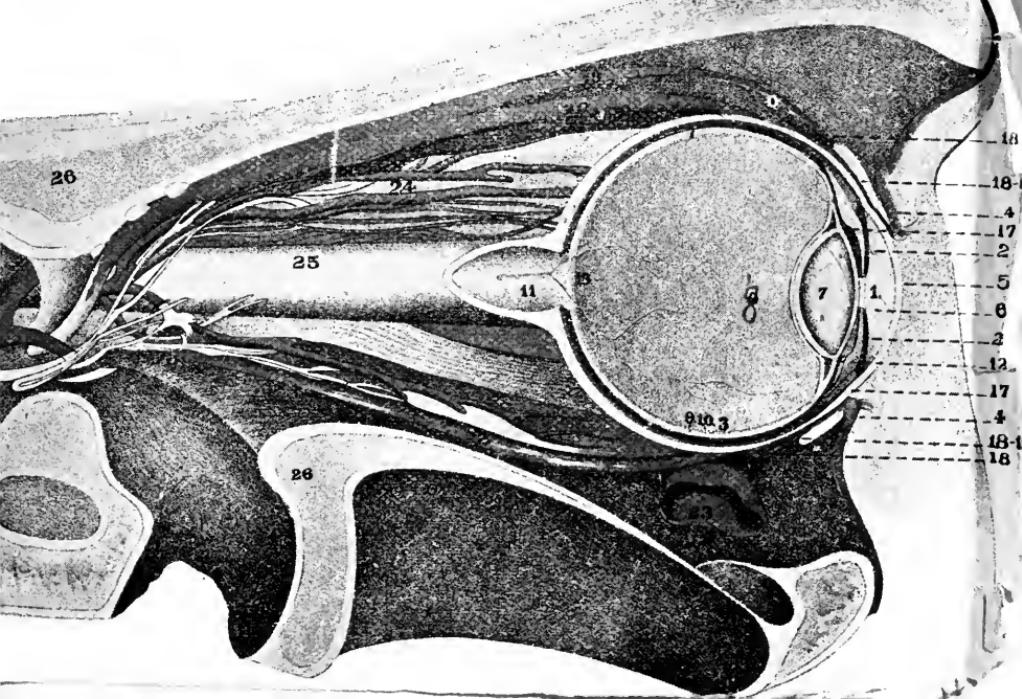
study of the following explanations will be of value to the dealer in optical g
business generally and are worth remembering.

ALITIES. It will be noticed that we show different qualities of the same styles of fra
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TEM OF STOCK NUMBERING. We have adopted the system of numbering stock
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t temple wire.	o indicates Flat eye wire, flat temple.
npin temple.	1 " oval eye wire, flat temple.
nd temple.	2 " " " round temple.
lf round temple.	3 " " " half round temple.
grooved lenses.	4 " beveled eye wire, flat temple.
ing temple, usual joints.	5 " for grooved lenses, riding temple.
lf riding temple.	6 " riding temple, usual joint,
ort end piece solid temple..	round eye wire.

passive, of this muscle. Alteration in density of the lens substance which would change its refracting power are so irregular in their nature that we are compelled to look upon lenticular regular astigmatism as due, probably without exception, to alteration in the curvature of the lens-surfaces, brought about by an unequal contraction of the ciliary muscle. I must confess to an inability to understand how some fibres of this circular muscle can contract through the same nervous impulse more strongly than others, and in just such a way as shall give a regular astigmatic form to the lens surfaces. Nevertheless, from the experiments and observations of Dobowozki, Javal, Laquer and others, the fact seems established, and we must accept it whether we can satisfactorily explain it or not.

§ 179. As in the other form of lenticular astigmatism so in this, the total astigmatism of the eye may be the result of its addition to or subtraction from the corneal astigmatism. Javal, as already stated, is of the opinion that in the majority of cases it is neutralizing in its effect, for in the greater part of the cases examined by him before and after paralysis of the ciliary muscle, the total astigmatism was greater after paralysis, and corresponded more nearly to the corneal astigmatism as determined by his keratometer. He thinks also that in youth many cases of corneal astigmatism are masked by an unequal action of the ciliary muscle. Laquer in his measurements found that in thirty-four cases where the total differed from corneal astigmatism, in nineteen the corneal was greater, and in fifteen it was less, and the differences were, without exception, in the lower grades. My own experience with the keratometer, so far, would seem to show that where there is a difference the corneal is, as a rule, greater than the total astigmatism (see table VI). Of course it is to be remembered in this connection, that in those cases where the total astigmatism is the greater the assisting lenticular astigmatism may be due to the oblique position of the lens, and that a paralysis of accommodation is essential to its positive exclusion.

§ 180. A partial action of the ciliary muscle may also be

due to a trauma which paralyses some of its fibres or the nerve filaments, just as we see an irregular dilation of the pupil as a result of the same cause.

§ 18 r.   It has been supposed that the operation for strabismus might exercise some influence on the corneal curvature, and Dr. Noyes has reported one case in which, after three strabotomies, the meridian of maximum curvature underwent a rotation of 25°.   The existence of such an action on the part of the operation for strabismus, has not, however, been demonstrated by keratometric measurements, and it is doubtful whether the corneal curvature is changed except, perhaps, in some very rare instance.

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CHAPTER XII.

Correction of Astigmatism.

§ 182. When a differential diagnosis of astigmatism is once accurately made, all the optical data are at hand for its correction. We have only to apply a cylindrical glass, which expresses by its refracting power the amount of astigmatism, with its axis in the direction indicated in the diagnosis, and the eye is rendered non-astigmatic.

A knowledge of the exact inclination of the axis of the cylinder is, therefore, of the utmost importance in writing orders for glasses, and it is necessary that we have some mode of measuring its degree. It can be done by drawing on a sheet of paper a series of radiating lines like Snellen's fan at intervals of 5°, and laying the trial frame straight on them with the center of the glass over the center of the radiating lines and noting with which line the axis marked on the cylinder corresponds, taking care always to read the degrees from the *patient's* left to his right.

A much more efficient and satisfactory method, however, is to have the degrees marked on the trial-frame itself. Several trial frames of this kind have been made, the best of which, in my opinion, is the one manufactured by Meyrowitz, of New York, and shown in Fig. 42. The frame is constructed to hold two lenses on each side, and one of them can be turned by means of the two little knobs shown in the drawing. When in an examination the position of the cylinder where vision is best is found, the axis of the cylinder marked on the glass points to the exact degree of inclination. This frame is further useful for determining the distance between the pupils and the

height of the nose, two important factors in the proper fitting of spectacles.

Fig. 42.
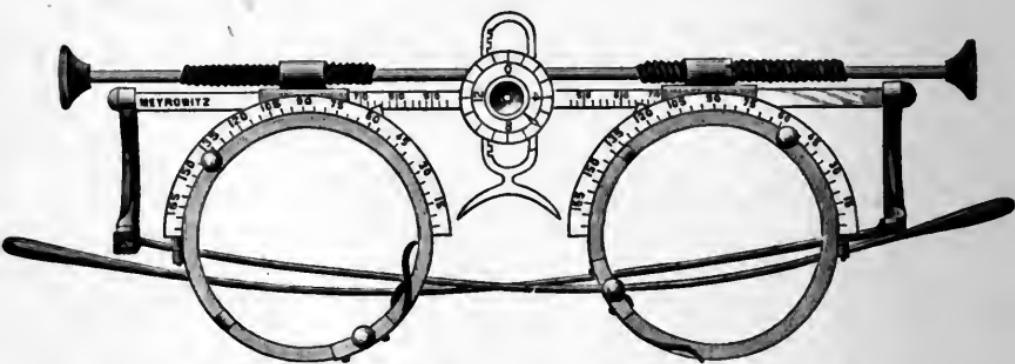
MEYROWITZ'S TRIAL-FRAME.[1]

§ 183. The first full account given of the correction of astigmatism by cylinders is that by the Royal Astronomer Airy, in 1827. For this reason and also to show that the correction was based on a scientific study of the optical condition, and to give the history of the term astigmatism as applied to this condition, I subjoin an account of this case taken from the last English edition of Mackenzie's classical treatise on the eye (London, 1854, p. 926).

"Mr. Airy discovered that in reading he did not usually employ his left eye, and that in looking at any near object it was totally useless, in fact the image formed in that eye was not perceived unless attention was particularly directed to it. Supposing this to be due entirely to habit, and that it might be corrected by using the left eye as much as possible, he endeavored to read with the right eye closed or shaded ; but found that he could not distinguish a letter, at least in small print, at whatsoever distance from the eye the characters were placed. Sometime afterwards he observed that the image formed by a bright point, such as a distant lamp or star, in his left eye, was not circular as it is in the eye which has no other defect than that of being near-sighted, but elliptical, the major axis making an angle of about 35° with the vertical and its higher extremity being inclined to the right. Upon putting on concave spectacles, by the assistance of which he saw distant objects distinctly with his right eye, he found that to his left eye a distant lucid point had the appearance of a well-defined line, corresponding exactly in direction and nearly equal in length to the major axis of the ellipse above mentioned. He found also that if he drew upon paper two black lines crossing each other at right angles and placed the paper in a proper position and at a certain distance from the eye, one line was seen perfectly distinct while the other was barely visible ; while upon bringing the paper

[1]The price of this trial-frame is $15.00.

nearer to the eye the line which was distinct disappeared and the other was seen well defined. All these appearances indicated that the refraction of the eye was greater in the plane nearly vertical than in that at right angles to it; and that, consequently it would not be possible to see distinctly by the aid of lenses with spherical surfaces. Mr. A. found, indeed, that by turning a concave lens obliquely, or on looking through a part near the edge, he could see objects without confusion; but in both cases the distortion was such that he could not hope to make any use of the eye without some more effectual assistance.

Mr. Airy's object was now to form a lens which should refract more powerfully the rays in one certain plane than those in the plane at right angles to it; and his first idea was to employ one whose surfaces should be cylindrical and concave, the axes of the cylinders crossing each other at right angles and their radii different. To show that this construction would effect the purpose, it is only necessary to imagine such a lens divided into two lenses by a plane perpendicular to its axis; thus it is easily seen that the refraction of the one will not be perceptibly altered by that of the other and the whole refraction will be a combination of the two separate refractions. The rays in one plane will be made to diverge entirely by the refraction of one lens, and those in the other plane by that of the other lens. This construction was then sufficient; but for the facility of grinding and for the diminution of the curvatures, it appeared preferable to make one surface cylindrical and the other spherical, both concave.

To discover the necessary data for the formation of the lens, Mr. A. made a very fine hole with the point of a needle in a blackened card, which he caused to slide on a graduated scale; then strongly illuminating a sheet of paper, and holding the card between it and the eye, he had a lucid point upon which he could make observations with ease and exactness. Resting the end of the scale upon the cheek-bone, and sliding the card on this scale he found that what was seen as a point when close to the eye, at a distance of six inches appeared a well-defined line inclined to the vertical about $35°$ and subtended an angle of (by estimation) $2°$; at the distance of $3^{1}/_{2}$ inches it appeared a well-defined line at right angles to the former and of the same apparent length. It was necessary, therefore, to make a lens which, when parallel rays were incident should cause those in one plane to diverge from the distance $3^{1}/_{2}$ inches, and those in the other plane from the distance six inches.

Having procured a sphero-cylindrical lens[1] of which the radius of the spherical measured $3^{1}/_{2}$ inches and that of the cylindrical surface $4^{1}/_{2}$ inches, Mr. A. found that he could read the smallest print at a considerable distance with the left eye as well as with the right. He found that vision was most distinct when the cylindrical lens was turned from the eyes; and as when distant from the eye the lens altered the apparent figure of objects by refracting differently the rays in the different planes, he had the frame of his spectacles made so as to bring the glass pretty close to the eye. With these precautions he found that the eye which he had once feared would become quite useless, could be used in almost every respect as well as the other."

*　　*　　*　　*　　*

"Having occasion 20 years after the first account of the malformation of his left eye was submitted to the Cambridge Philosophical Society (1849) to explain that a change had happened in the state of the eye, Mr. Airy took an opportunity of men-

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[1] This glass was made by Fuller, of Ipswich.

tioning that as the nature of the effect of that malformation was, that the rays of light coming from a luminous point and falling on the whole surface of the pupil did not converge to a point at any position within the eye, but converged in such a manner as to pass through two lines at right angles, the Rev. Dr. Whewell had affixed to this phenomenon the term *Astigmatism*."

All knowledge acquired since this account was published has added nothing to the theory of the astigmatic condition or to the philosophy of its correction.

Astigmatism, however, had been corrected independently, it would seem, by a number of individuals during the years between Young's discovery of the condition and the beginning of the new era inaugurated by the investigations of Knapp and Donders. The optician Cary informed Young that he had found many myopes who saw better when their concave glasses were tilted. It also appears that the painter Cassus noticed some peculiarities in the work of his master, Gros, at Paris, in 1818, which he referred to a defect in sight, and that this defect was corrected by cylinders made by Guscipi, of Rome, in 1840-44, and these were afterwards duplicated by Soliel, of Paris.

In 1852 Goulier sent to the French Academy of Science a sealed communication with the request that it be not opened until 1865. When it was examined it contained a good account of astigmatism (given in Abstract in Javal's article on the history and bibliography of astigmatism in *Am. d'Ocul.*, *1866*) and manner of its correction by cylinders. A Swiss priest, Snyder, of Luzerne, also detected in himself an astigmatism which he corrected by cylindrical glasses in about 1849.

Dr. Isaac Hays in his American edition of Laurence on the Eye (1854), relates in full a case which had been successfully fitted with cylinders by the Philadelphia optician, McAllister, in 1825, and two others which had in that year (1853) come under his own observation in which the same optician had improved vision by the same means; but no accurate account of these last was given.

§ 184. Easy, however, as it may appear theoretically to correct astigmatism, when we come to deal with the question practically it is not always so simple as it seems. We are dealing here in part with positive science and it is essential that our methods should be exact if we expect our results to be perfect.

§ 185. In the first place, it is necessary that the diagnosis be in all respects correct. We must not only know the inclination of the faulty meridian to within 5° (or even less in some cases) but the exact state of the refraction in each meridian separately. To obtain these in the majority of cases, as we have already seen, requires the expenditure of much time and patience, and the practitioner who hopes for uniform success and satisfaction in his astigmatic cases must grudge neither. In many cases it is only by examining and reëxamining and

testing and proving by many methods that the true condition is revealed, and sometimes it is necessary to give glasses to be worn for a time, in order that the close observation of the patient may throw some light upon an obscure point.

§ 186. The methods of dealing with astigmatism after a correct diagnosis has been established may be best shown by some cases illustrative of the various forms to be dealt with.

CASE I. It has been shown by the various tests that there is simple myopic astigmatism in the vertical meridian. With —2 axis 180° V=⁴/₄ and Snellen's fan is uniform and clear. You order:

L.	R.
—2 axis 180°.	—2 axis 180°.

And if he is a young man of 18 and a student, or employed where he uses his eyes constantly for close work, you order him spectacles which he is to use constantly. If he thus early makes them a part of his eyes he places himself in the condition of an emmetrope and runs much less risk of trouble in future.

Fig. 43.
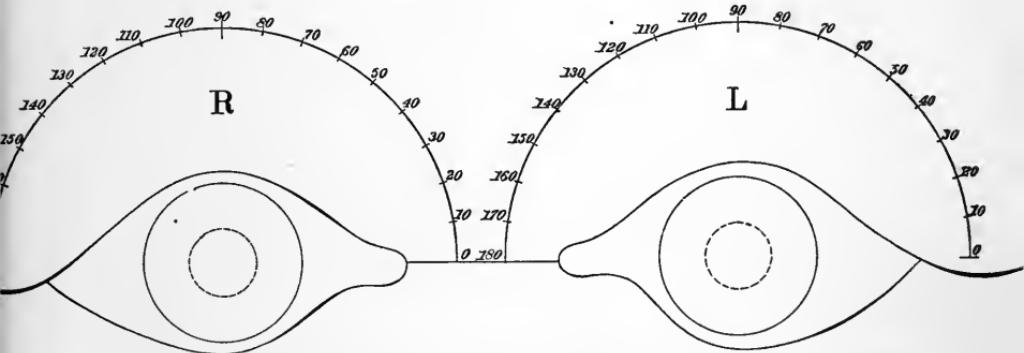
DIAGRAM FOR RECORDING ASTIGMATISM AND FOR ORDERING GLASSES, REPRESENTING THE PATIENT'S EYES AS SEEN FROM THE FRONT.

We should always in recording diagnoses and ordering glasses follow a uniform system in order to avoid a troublesome confusion and liability to error which would otherwise inevitably occur. *We must always consider the glasses as the patient looks through them,* and in counting the degrees of inclination of the axis of the cylinders, proceed always from *his left to his right;* and it is well to have this diagrammatically represented as in Fig. 43. Such diagrams are also useful for

recording graphically many other morbid conditions and injuries of the anterior portion of the eye-ball and of the lids. The method of using this diagram is shown in Figs. 20 and 21.

CASE II. A lady of 30 is found to have compound hypermetropic astigmatism with V=⁴/₁₂. She is unable to read the evening paper with comfort and her fine needle-work tires her eyes even in the daytime. There is a general H of 1.5 with H. astig. of 0.75 axis at 45° in L and at 135° in R. With this correction V=⁴/₄ and No. 1 is read with ease and comfort at ten inches. She is ordered :

<div align="center">

L.   R.

+ 1.5 ⌒ + 0.75 45°.   + 1.5 ⌒ + 0.75 135°.

</div>

You insist upon the importance of her wearing these glasses constantly in order to save her from a possible future break-down; but she objects so strongly that you feel perfectly certain she will not do it, particularly as she says her distant vision is as good as she cares about. She also rebels at spectacles and wants to know why nose-glasses won't do as well.

This is the battle that goes on in the consultation room daily, and the surgeon will commonly find it wise to effect some sort of compromise, particularly at the beginning. It is much more satisfactory and much less trouble to the patient to have cylinders set in spectacle frames since the axes are there always at the same angle and require no adjustment; but young women are, as a rule, so opposed to wearing them, particularly in public, that they will often suffer rather than use them. They find nose-glasses less objectionable, and most opticians can now fit cylinders in them so that they can be adjusted properly on the nose with little trouble after patients have become accustomed to their use. You therefore have one pair of glasses set in spectacle frames for use at home when she is doing continuous work or reading, and another pair in *pince-nez* for reading the hymns in church, the programmes at concerts or the theatre, for picture galleries, shopping, etc. In the course of time she will find her distant vision so much improved by the nose-glasses that she will wear them pretty constantly, and finding in the end that the spectacles are much less troublesome will most probably come to substitute them for the nose-glasses for all purposes.

§ 187. Another condition which comes in as a complication is that of *presbyopia*. When the accommodation begins to fail its effect must be taken into consideration, since the same glasses will no longer do for far and near vision.

CASE III. A gentleman of 48 complains of his inability to read with comfort in the evening. His distant vision has been sufficiently good for him as a professional man, though he has always considered himself somewhat "near-sighted." On testing it is found that there is myopic astigmatism of ¹/₄₈ in the vertical meridian, and correction brings V up from ²⁰/₅₀ to ²⁰/₂₀. With — ¹/₄₈ axis 180° for both eyes, he is not able to read No. 1 at 12 inches, but with a +¹/₄₈° placed in front of the cylinders he reads it with facility at from 10 to 18 inches. Instead however, of ordering

him a compound lens with a $-\frac{1}{48}°$ on one face and a $+\frac{1}{48}°$ on the other we simply write for

L. R.
$+\frac{1}{48}.90°$ $+\frac{1}{48}.90°$,

thus giving the optical result of the combination, for the spherical $+\frac{1}{48}$ neutralizes the $-\frac{1}{48}$ cylindrical axis at 180° leaving a $+\frac{1}{48}$ action at 90°, thus practically rendering the eye myopic $\frac{1}{48}$ in all its meridians. This relieves the accommodation sufficiently for a time and gives satisfaction.

CASE IV. A lady, 50 years of age, has had great difficulty in getting glasses for reading. She has gone from one optician to another and has accumulated a store of glasses of various kinds, but they are all unsatisfactory, and she has at last settled down to the belief that there are no glasses that will fit her eyes, particularly as she considers them "weak," her distant vision never having been good. She has at last, however, been pursuaded to have her eyes examined by an oculist and you find after a careful investigation that there is compound hypermetropic astigmatism. In L $+1 \bigcirc +1.5$ axis 70° gives V$=\frac{4}{6}$; in R $+2 \bigcirc +0.75$ axis 60° gives V$=\frac{4}{5}$, and no other glasses or combinations do better.

With both eyes corrected V$=\frac{4}{4}$ nearly, and with them the fan is clear and uniform. Javal's keratometer verifies the degree of astigmatism and the direction of the principal meridians.

This case offers departures from the usual in that the general ametropia is different in the two eyes (anisometropia), while the degree of astigmatism is not the same for each eye, and the direction of the faulty meridians is not symmetrical. When the meridians are oblique, as a rule, they stand at the same angle outward or inward for each eye—very seldom is one outward and the other inward, a fact which would seem to point to a common formative cause at work for the production of the malformation.

With these glasses, however, she cannot read; her presbyopia has to be corrected. It is found that by adding $+2.5°$ to the distance glasses she can read the finest print with ease. You therefore order correction for distance, and give her the following for reading:

L. R.
$+3.5 \bigcirc +1.5^{cy}$ axis 70°. $+4.5 \bigcirc +0.75^{cy}$ axis 60°.

With her glasses life assumes a new and decidedly more cheerful aspect. She can see at a distance as she never could before, and the long hours of the evening are passed pleasantly in reading, something that before was impossible.

CASE V. This is a young man of 21. His vision is very bad, being only $\frac{4}{60}$, and a long and careful examination with sphericals and cylindricals fails to bring it up to more than $\frac{4}{12}$ and he is benefited by both $+$ and $-$ lenses. The keratometer of Javal showed an astigmatism of 4.5 D at 180° in L, and the same at 25° in R. In the trial by lenses, however, the answers and statements are so contradictory and uncertain that it is considered expedient to paralyze the ciliary muscle in order to get rid of the interference of the accommodation. So a drop of a 4 gr. solution of atropia sulph. is ordered to be put into each eye four times a day for three days when he is to return for another investigation. His eyes are examined now by the ophthalmoscope, and in the left eye the fine vessels running horizontally over the sides of the disk are seen in the erect image only when -2.75 is brought behind the hole in the

mirror. Through this lens all the other vessels are dimmed in outline and the finer vertical vessels near the macula are not distinguished at all. These last are seen only when a +2 is behind the mirror. This gives at once a clue to the condition. On trying by the inverted method the disk contracts in its horizontal diameter and enlarges in its vertical diameter as the lens is removed from the eye, and the vertical diameter contracts and the horizontal enlarges as it is brought closer to the eye, and the ovals are vertical and horizontal. There can be no opinion now but that the case is one of mixed astigmatism, and proceeding, on the indications thus furnished, to a reëxamination with glasses we soon find that with —2.75° axis 180°, combined with +2° axis 90° V=⁴/₅. In the R eye the examination with the direct ophthalmoscopic method is not so satisfactory, but the disk appears an oval standing obliquely, and when a — 3.5 or —4.5 is used behind the mirror, those parts of the vessels running obliquely upward and inward are most distinct; and when +1 or +1.5 is used those parts of the vessels running upward and slightly outward are sharpest in outline. In the inverted image the disk, instead of being a vertical oval when the lens is removed from the eye, is oblique with its top inclining outward. Turning now to the directions of the corneal meridians, as given by the keratometer, we find that the meridian of greatest refraction has its axis at 25°, and the meridian of least refraction its axis at 115°. A very few trials with glasses now show us that with —3.5° axis 25° $\supset$ +1.5° axis 115° V=⁴/₆.

In ordering glasses for mixed astigmatism two plans can be followed. One is to have one surface of the lens ground as a cylinder, giving correction to one meridian, and the other as a cylinder correcting the other meridian, with their axes at angles—crossed cylinders as they are called. For the above case, therefore, we write:

<div style="text-align:center">

L.   R.

—2.75 180° $\supset$ +2, 90°.   —3.5, 25° $\supset$ +1.5, 150°.
</div>

This is the form of astigmatic lens which, as we have seen, Airy first conceived for his own eye, and it is considered by many to be the best in some particulars. Among other advantages it is thought to give a flatter field. But it is an expensive lens to manufacture for the trade, and there is more risk of an error in the direction of the axis than in the other form. The other plan is to convert it into a sphero-cylindrical lens, in which there is only one cylinder to be made. This is done in the following manner:

If we take, for the left eye in above case, a —2.75 plano-spherical and grind on the other side a +4.75 cylinder we have practically the formula given above, for the +4.75 cylinder overcomes

the —2.75 in the meridian corresponding to its curvature and gives in addition a + cylindrical action of (4.75–2.75) 2 D, while the — action of 2.75 of the sperical lens in the meridian at right angles to it is unaffected. Applying the same principle to the construction of the lens for the right eye, we would order:

$$\text{L.} \qquad\qquad \text{R.}$$
$$-2.75^{s} \supset +4.75^{cy}\ 90^{\circ}. \qquad -3.5^{s} \supset +5^{cy}\ 115^{\circ}.$$

This method can be modified in many ways and often to the advantage of the patient's pocket. Suppose, for example, that both eyes of the patient were affected with the mixed astigmatism of the left eye of the case just considered. When he arrived at 50 or 53 years his reading glasses could be made in the form of simple cylinders +4.75, 90°, thus rendering the whole eye myopic 2.75 D, and relieving his accommodation to the desired extent. Plane cylinders are not so expensive as spherocylinders, and we should not be above considerations of this kind, particularly for persons of limited means who lose or break their glasses frequently.

§ 188. It is a fact, however, which occurs with an unfortunate frequency in our clinical experience, that the glasses which give perfect optical correction, particularly if this is determined under atropine, cannot be worn with comfort and sometimes not at all. Such cases are to be found, as a rule, in persons who have passed their youth with their anomaly uncorrected. Of this condition the following case is an illustration.

Case VI. Mrs. P., æt. 30, has suffered from asthenopia and bad vision for a long time, for neither of which has she found any glasses beneficial. On examination I found V=^{4}/$_{18}$ in L, ^{4}/$_{24}$ in R. Plus glasses gave no improvement, but minus spherical glasses from 1 to 3 did increase the visual acuteness somewhat. With —2.25 axis 180° V=^{4}/$_6$ in L, ^{4}/$_9$ in R. An opthalmoscopic examination by the direct method showed no myopia in the vertical meridian, but on the contrary, a hypermetropia in the horizontal meridian. This discrepancy in the findings by the subjective and objective methods being to me always an indication for the paralysis of accommodation, I ordered her to apply a 4 gr. solution of atropine three times a day and report for reëxamination at the end of three days. Under the mydriatic it was found that V=^{4}/$_{60}$ in R, ^{4}/$_{36}$ in L, and with +2.5° axis 90, V=^{4}/$_9$ in R and ^{4}/$_6$ L. This condition was confirmed by the ophthalmoscope. As is my custom I allowed the effect of the mydriatic to pass off before ordering glasses. At the end of ten days when the pupil had regained its normal size, I found that the + cylinders gave

her no improvement for distant vision, but on the contrary, the concave cylinders did. She was given, however, $+2.5^c$ 90° for each eye, with instructions to gradually accustom herself to their use. These instructions she followed faithfully for three months, but at the end of that time found herself in no better condition than before. Her vision for distance had not improved, and while she could see better for close work with them, her asthlnopia was not relieved ; in fact, the discomfort was greater with the glasses than without them. She could see well with -2.5^c 180°, and she was now ordered these for experiment. Distant V was more satisfactory, but she could not use them for near work. She was in despair, when happening one day to pick up her husband's glasses that I had prescribed sometime before for a H of 0.75 in the horizontal meridian, she found comparative comfort in reading, and these are the only glasses that we have found up to this time which are of any material advantage. We hope, however, in course of time to be able to educate her up to the use of full correcting glasses, by gradually increasing their power.

The case was first examined before I had any means of keratometric measurement, but lately I examined her with Javal's instrument, and found in both $R=8^1/_2$ mm. at 180° and $8^1/_4$ at 90°, with a crossing of the bands of $2^1/_2$ steps at 180°, thus verifying the diagnosis made by the other methods.

Cases so extreme as this are not common, but lesser degrees are frequently met with. I can only account for them on the supposition that the eyes have been accustomed for so long to the astigmatic state as to make the abnormal, in a certain sense, the normal condition and that the cerebral center for vision resents an interference with the established order of things. It must be remembered, in this connection, that in dealing with the human eye we have to do not with an optical instrument alone, but with an organ of sense as well. All of our senses are, in a measure, affected by education, and after a certain habit has been once firmly fixed it is with difficulty changed in any important particulars. It is a fact now generally acknowledged that when strabismus has existed for a great while, binocular vision is not obtained even after the optical axes are correctly placed by an operation.

It has seemed to me, therefore, not only unwise, but useless to attempt to force eyes back to our conventional standard, and to insist on patients wearing glasses giving full correction, when a fair trial has proven their unsatisfactoriness. Under these circumstances, it would appear best to find the glass, generally a weak one, which gives most comfort and tentatively increase the strength.

The case just related also well demonstrates the condition

commonly called " spasm of accommodation." There was a much higher refraction manifest before the action of the atropine than after, and this is usually attributed to a spasm of the ciliary muscle. If by " spasm " is meant a permanent tonic contraction of the muscle, the term is certainly misapplied, since when there is nothing to call the accommodation into play the muscle is relaxed, as shown by the direct opthol-moscopic examination. The unusual contraction of the ciliary muscle in this case is, in my opinion, a voluntary act, and is due to the fact that the patient from custom or preference has always used her anterior focal plane, which would necessitate such a contraction of the ciliary muscle as shall convert a H. astig. axis 90° to a M. astig. of the same degree axis 180°. The use of this plane has become a fixed habit with her and the probabilities are that, at her time of life, it can be broken up with difficulty or not at all. Such difficulties as these are much less frequent in younger people who can more readily adapt themselves to altered conditions.

Another aggravated case of this character is the following :

CASE VII. A lady, 42 years old, had been treated for asthenopia by Dyer's method for some months, without, however, the discovery being made of any refractive anomaly. A Philadelphia surgeon, to whom she applied later, worked out, under atropine, a high degree of astigmatism which I found to agree essentially with that diagnosed by myself. The data obtained by the keratometer of Javal were: L, 15° $R = 7^3/_4$ mm. 185° $= 8^1/_4$ mm. with a crossing of $3^1/_2$ steps; R. 75° $R = 8^1/_4$ mm. 160° $= 7^1/_2$ mm. with a crossing of $4^1/_2$ steps. V without glasses $= ^4/_{60}$ in L., less than that in R. With -3.5^{ey} 15°, L V $= ^4/_{18}$, with -4.5 70° R V $= ^4/_{18}$.

The Philadelphia surgeon had ordered correcting glasses, with instructions to persevere in their use. This she had done, but the longer she wore them the more uncomfortable they became. They made her so dizzy that she was utterly unable to wear them in the street, and she could not read with them at all. She was then ordered glasses which would combine a $+1.5$ with the correction above indicated, with the hope that thereby she would be able to use her eyes for near work at least, and in time work her way gradually to full correction for distance. It was because she found that after several months' trial it would not be possible to use either of these with benefit or even comfort, that application was made to me. With this experience before me I had not much hope of benefiting her by means of glasses, particularly as I suspected that a large part of the asthenopia was nervous. I gave for experimental use, L $+3.5$ 105°, R $+4.5$ 160°, to be used for near work. She could read with comfort for a somewhat longer time with these than without them, but the benefit was not at all encouraging.

§ 189.   Do cylinders give an absolute correction of the astigmatic condition?   They can not, from the fact that the refraction of an elliptical surface cannot be neutralized by a surface which has equal radii of curvature.

Fig. 44.
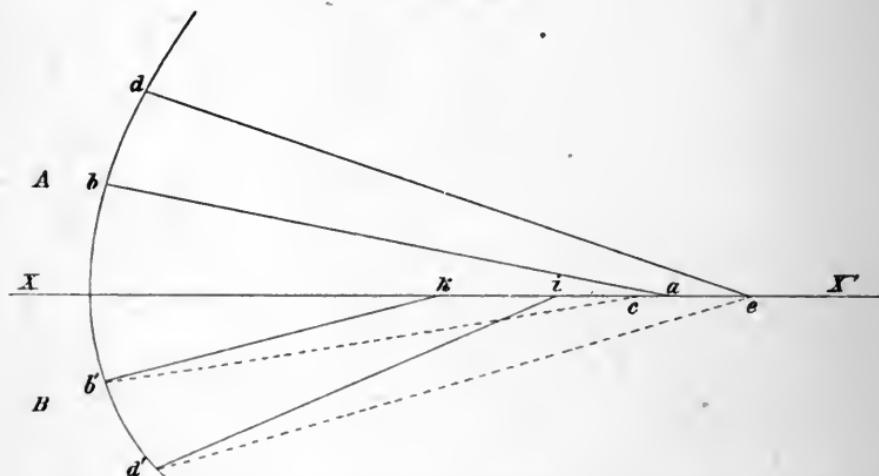
EFFECT OF A CYLINDRICAL LENS ON THE REFRACTION OF THE SHARPER END OF AN ELLIPSOID WHEN THE PERIPHERAL RAYS OF THE TWO MERIDIANS ARE BROUGHT TOGETHER.

§ 190. Where the cornea, as it usually does, represents a triaxial ellipsoid, we have a different set of conditions according to the special character of the curvature; and the action of cylindrical lenses on the refraction of the principal meridians will not be uniform in all cases.

Let us take, as an example, that form in which the cornea represents the sharper end of an ellipsoid with three unequal axes. It is plain from what has been demonstrated in Chapter II that the meridian of greater curvature, should it pass a certain point, will suffer from the greater aberration. A in fig. 44 represents the meridian of less, and B the meridian of greater curvature.   In A, the peripheral ray d crosses the principal axis XX′ at e and the more central ray b at a, while in B the corresponding ray d' crosses at i, and b' at k.   If we place a cylindrical lens before the refracting surface with its curvature corresponding to the meridian B, and of such

strength that the peripheral ray d' is carried back and made to cross the axis in the same point e as the peripheral ray d of the meridian A, the relation between k and i, though they are both carried back from their original position, remains unaltered at c and e, since the regular refraction of the cylinder does not counteract the aberration of the elliptical surface. The result would be that the rays crossing at a and c, would form figures of diffusion on the focal plane passing through e.

Fig. 45.
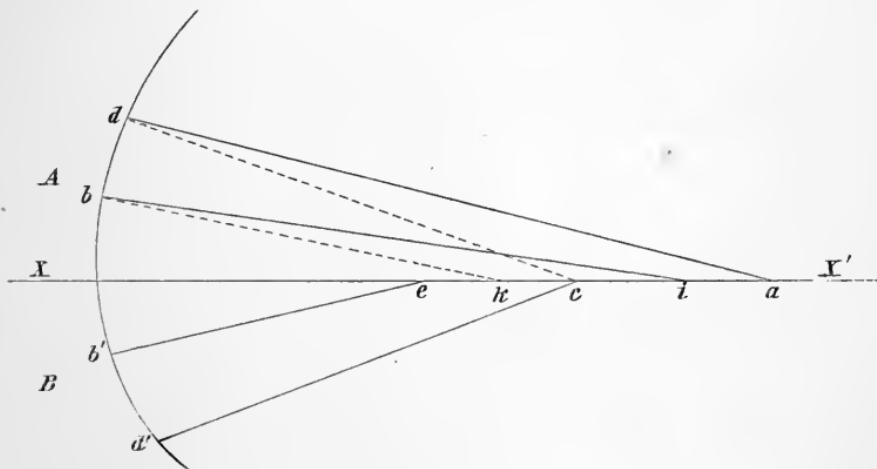
ANOTHER EFFECT OF A CYLINDRICAL LENS ON THE REFRACTION OF AN ELLIPSOID WHEN THE PERIPHERAL RAYS OF THE TWO MERIDIANS ARE BROUGHT TO-GETHER.

If we bring the more peripheral rays, d and d', of the two meridians, A and B, to cross at the same point c, moving them forward from a, i, as in fig. 45, we have the same result; for the central rays b, b', which cross the axis at e and k, would form figures of diffusion on the focal plane passing through c.

§ 191. We would have, of course, an analogous state of affairs in dealing with the blunter end of the ellipsoid; for while it would be possible, by means of a cylindrical lens, to bring corresponding peripheral or central rays to cross the axis at the same point, it would not be possible to bring *both* the cen-

tral and peripheral rays to cross it in one point; and if we should have to deal with a surface in which one meridian represented the blunter end of an ellipse, while the other represented the sharper end, the diffusion figures would be still more confusing. Fig. 46 represents such a surface where the peripheral rays, d, d', are brought, by means of a cylinder, to cross the axis at the same point e. The more central ray b of the flatter ellipse A, will cross the axis at c, *behind* the focal plane, passing through e, while the more central ray b' of the sharper ellipse B, will cross it *in front* at a, thus forming two sets of diffusion figures.

Fig. 46.
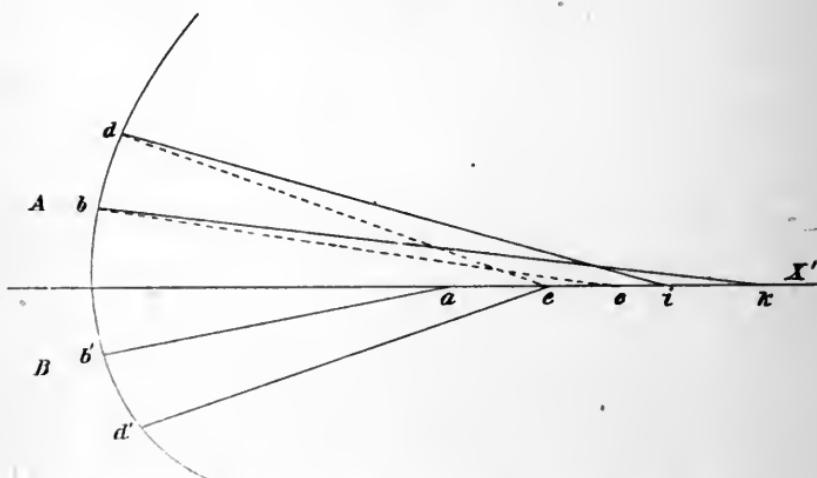
REFRACTION BY THE BLUNTER AND SHARPER ENDS OF AN ELLIPSOID CORRECTED BY A CYLINDER.

§ 192. Under any of these forms which the cornea may assume,[1] the retinal image must have its distinctness of outline impaired by the circles of diffusion which fall on it.[2] This dif-

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[1] In all the measurements that have been made up to the present time, the cornea has never been found to assume in either of its principal meridians the form of the blunter end of ellipse, but we see no reason to doubt the *possibility* of such an occurrence.

[2] We do not take into consideration here the rays passing through the intermediate meridians. These require a separate investigation.

fusion being greater in the higher than in the lower forms of astigmatism, we should expect to find the visual acuteness, after all possible correction, less in the former, and such I believe is the experience of all practitioners. We should also expect that the vision of astigmatics, after correction, would be less than that of myopes and hypermetropes of the same grade after their neutralization by spherical lenses.

As a matter of statistics, I find that out of about 2,000 astigmatic eyes of all degrees, only about $1/_{10}$ have $V = 1$, after the best possible correction.

§ 193. It is apparent that this aberration in the principal meridians will be greater, the greater the angular aperture— which in the eye would be represented by the pupil—and consequently the larger the pupil the larger the figures of diffusion, and the more indistinct the retinal image.

This is an additional reason for not accepting the examinations made under atropine as absolute, since the effect of the diffusion images on the retina will be different with a large pupil and with one of normal size, and this difference is likely to be strongly felt in the final correction.

§ 194.   Another point in the action of cylinders to be taken account of in this connection is their influence on the position of the nodal points in the meridian of their action.   While the cylinder in correcting the abnormal refraction brings the focal points of the two meridians together approximately, it advances the nodal points of the meridian it affects when it is convex, and causes them to recede when it is concave.   Now, the size of the retinal image is governed by the position of the nodal points in relation to the retina.   This image is larger for the same object the farther the nodal points are removed from it.   We should, consequently, expect to find a diminution of the image on account of the recession of the nodal points, when a concave cylinder is used, in the direction of the faulty meridian, and an enlargement of it in the direction of the hypermetropic meridian when a convex cylinder is used.   As a result, there would, theoretically, be an enlargement of an object in the direction of the meridian corrected by a + cylinder, and a

diminution of it in the direction corrected by a — cylinder.

§ 195. How low a degree of astigmatism it is necessary to correct? This is not purely a question in optics, but one in answering which many considerations must enter.

Cylindrical glasses should not always be prescribed simply because distant vision as tested by the test types is thereby rendered better, for some persons with an astigmatism of 0.75 D have a sharpness of sight for distant objects, on account of their better interpretation of retinal impressions, superior to some whose eyes are emmetropic. If such persons are satisfied with their distant vision, and do not suffer, no good can result from forcing on them the constant use of glasses. The use of glasses is a great inconvenience, many persons are strongly prejudiced against them, and they are more or less expensive. The wearing of weak glasses should, therefore, not be made imperative, if decided objection is urged, unless the surgeon is satisfied that undoubted benefit will follow their use.

But when there is a complaint of asthenopia, even when the eyes are not used at close work, and particularly in nervous women, the correction of even low degrees of astigmatism is usually attended with great benefit. The constant use of cylinders of 0.5 D is often sufficient to transform misery into comfort. And in almost all cases the addition of a 0.5 D cylinder, where it is required, to presbyopic glasses makes reading much more comfortable, in the evening especially, or where close application for a considerable time is necessary.

It occasionally happens, too, that the correction of astigmatism as low as 0.25 D is found very beneficial. Such cases are usually found in persons whose nervous systems are below par, and on restoration to health the glasses can be laid aside.

When the amount of astigmatism after cataract extraction exceeds 1 D there is always an advantage from its correction; for less degrees it is hardly worth while.

§ 196. We would call attention here to a fact in the correction of astigmatism which has not yet met with a satisfactory

explanation, and that is the improvement in vision given by the tilling of a spherical lens superior to that afforded by a cylinder equivalent to that amount of inclination of the spherical. It is a matter of common observation that persons operated on for cataract often see better when their spectacles are held obliquely. This difference in effect may be due to some difference of action on the rays passing through the intermediate meridians, a subject which has not been fully examined into on account of the great difficulty in obtaining formulæ of general application.

It may also be stated, in this connection, that some astigmatics can correct their anomaly appreciably by making pressure at the proper place on the sclera to give the correcting curvature to the cornea in that meridian.

§ 197. When a prescription for glasses has been given, the case should not be summarily dismissed. The patient should be instructed to bring the glasses back for examination, for it is of the greatest importance that the optician shall have followed strictly the instructions of the surgeon. An approximation to the glasses ordered will by no means suffice. A small error in the number of the lens, or a change of 2° or 3° in the position of the axis of the cylinder will often mar an otherwise good result. The mistake, moreover, may not always be the opticians. In the hurry of many examinations the surgeon himself may have put down the wrong number or the wrong degree, or put L for R. The necessity of some method of proving glasses is therefore apparent.

§ 198. There are several methods by which this may be done but the one which is most rapid and best adapted to the consultation room is that of *neutralization.*

When a + and — lens are placed together the action of the one, as is well known, tends to neutralize that of the other, so that the combined power of the two is always equal to the *difference* in their refraction. When the strength of the two is the same, their optical action will be nil, the same as a bit of plane glass. When, therefore, we find, for example, a — lens whose power we know which completely neutralizes a + lens,

the power of which we did not know, the number of the one gives us the number of the other.

How shall we know when the one neutralizes the other? One very simple method is that of watching the paralactic movement of objects through them.

When a *convex* lens is moved back and forth a few inches before the eye, and at right angles to the optical axis, objects seen through it are observed to move in a direction *opposite* to that of the lens. Through a *concave* lens the movement is in the *same* direction as that of the lens.

Let us have, for example, a — lens whose number we are ignorant of and the exact refracting power of which we wish to find. If the large types of Snellen are very indistinct through it, we know at once that it is a strong glass, and begin by placing on it a strong + glass, say No. 4. With this most of the letters of the test-types are seen, but there is still a movement of objects "with" the lens. With a + 5, there is a slight movement of the letters " against " that of the lens, showing an excess of + action. No. 4 is, therefore, too weak, and No. 5 too strong. We now try + 4.5 and find that in whatever direction the combined lenses are moved objects remain stationary. It is possible by this test to tell to within 0.25 D the power of any spherical lens, and this is sufficiently accurate for all practical purposes.

§ 199. The question becomes somewhat more complicated, however, when cylinders are to be dealt with, because in them we have to determine not only the strength of the lens, but also the direction of its axis. In this examination we employ the fan of Snellen.

Example : The lens whose power is to be determined, is a sphero-cylindrical convex. Through it the whole of Snellen's fan is indistinct. But by holding in front of it concave sphericals, one after the other, a lens is finally found which shows the line at 20° clear and sharp. The concave lens which gives the neutralization is of course the *weakest* one through which this line appears with clearly defined edges and which gives no paralactic movements. If this is — 2.5, then we know that the

meridian of the sphero-cylinder whose axis is at 20° is + 2.5.
But with this, the lines to the right of the center of the fan are
still blurred and offer paralactic movements. We now try the
addition of concave cylinders with their axis at 110°, that is in
the direction of the blurred lines, until one is found which
renders the fan uniform and clear. When these lenses are
moved together there will be no paralactic action if the correc-
tion has been complete. Should there be a movement of objects
it must be noted whether it is in the direction of the cylinder's
axis or at right angles to it or in both directions, and of course
also whether "with" or "against" the lenses. If there is a
movement when the lenses are moved parallel to the direction
of the axis of the cylinder, then the spherical lens is not right.
If there is no movement in this direction, but only at right
angles to the axis of the cylinder, then the cylinder is at fault,
and the lens in either case will have to be increased or dimin-
ished in power according to the direction of the movement, un-
til one is found with which all movement ceases.

§ 200. Dr. E. Gruening, of New York,[1] employs a method of
"simultaneous contrast" which he has found very convenient,
and which he prefers, as being more accurate, to the one just
described.

When a narrow stripe of a marquetry floor or carpet is
looked at through a convex lens, that part seen through the
lens appears wider than the part lying to either side of it ; when
viewed through a concave lens it appears narrower. As the
margins of the stripe inside and outside the lens are seen at the
same time coming up to the edge of the lens, a small differ-
ence in magnitude can be readily detected. In applying this
principle to the testing of spherical glasses we have only to hold
the lens to be proven horizontally parallel with the floor and,
looking down through it at the stripe, observe whether the part
seen through the lens is larger or smaller than that lying out-
side of it. If it is larger the lens is convex and we apply con-
caves, as in the preceding experiment, until the stripe is con-

[1] Verbal communication.

tinuously of the same size from both sides through the lens. If the part seen through the lens is narrower, there is an excess of minus action, and we apply convex lenses until one is found which gives a uniform stripe.

The same principle holds good, of course, for cylinders when their axes lie in the same direction as the stripe. In the meridian perpendicular to the stripe they act, to all intents and purposes, as sphericals, and may be so considered. If, therefore, the stripe is seen through the cylinder, in this position, to be enlarged, there is a plus action, and it is corrected by placing a concave cylinder on it with its axis coinciding with the stripe and the axis of the lens to be tested; if it is diminished in size there is an excess of minus action, and the correction is made by convexes, applied in a similar manner.

But the method is likewise useful in determining the direction of the cylinder's axis. The portion of the stripe seen through the cylinder and the portions outside of it do not run in the same direction except when the stripe and the axis of the cylinder coincide or are at right angles to each other. Any deviation from these two positions causes the two portions to lie at angles to each other. Knowing the angle at which the axis of the cylinder should be, and finding the angle at which it is necessary to place the lens to be determined in order that the parts within and without the lens fall in the same direction, we here have the necessary data for determining whether the axis is properly placed.

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CHAPTER XIII.

IRREGULAR ASTIGMATISM—CONICAL CORNEA.

§ 201. Regular astigmatism, as we have seen, is a condition of the eye in which its refraction approaches that of a triaxial ellipsoid where the principal meridians are ellipses and at right angles to each other. All other departures from a strictly spherical refraction are classed under the general term—irregular astigmatism.

§ 202. IRREGULAR ASTIGMATISM can have its seat in either one of the refracting media of the eye and may, consequently, be *lenticular* or *corneal.*

§ 203. With very rare exceptions all eyes are affected with a certain amount of irregular astigmatism, but when it is not sufficient to reduce V below $^{20}/_{20}$, it is considered as normal.

Normal irregular astigmatism is for the most part lenticular and is the result of a want of homogenity in the lens substance.

This want of uniformity of structure is due, mainly, to the manner of the lens's growth. The development of the lens is not symmetrical in its entirety, but in parts or sectors independent of one another and these are afterwards joined together to constitute a whole. It is seldom that the union of these sectors is so perfect as to leave no trace of their separate existence.

§ 204. This peculiarity in the construction of the lens is demonstrated by dissection and by entoptic experimentation.

Anatomical investigation shows not only a gradual increase in the density of the lens substance from the circumference towards the center, but also the remains of its sectorial evolu-

tion.   Fig. 47 is a meridional section of the lens of a young infant in which this sectorial character is well represented. This structure of the lens is also often apparent in the living eye under oblique illumination, and particularly so in some cases of cataract, where the line of union of two adjacent sectors is plainly visible.

§ 205. The refractive unevenness of the lens caused by its anatomical structure is shown by means of a simple entoptical experiment.   Make a fine point of light by holding a strong convex lens, such as the ocular of a microscope, at ten or

Fig. 47.
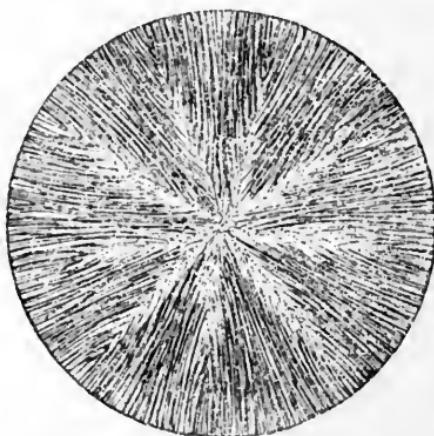
THE SECTORIAL CONSTRUCTION OF THE HUMAN CRYSTALLINE LENS.

twelve feet from a gas jet.   In this manner a fine pencil of rays is obtained which we can use for casting on the retina shadows of such opaque objects in the refracting media of the eye as lie in its path, while variations in density will manifest themselves by brighter or darker lines or spots.   As this bright focus is approached to the eye we see first, the shadows cast by the objects on the surface of the cornea, such as tears, meibomian secretions, etc.   As it is brought closer, the diffraction of the pupillary edge of the iris is seen, and then the inequalities of the lens structure, those on the anterior face being first brought to view, afterwards those at the center and on the posterior face

successively ; and finally the objects in the vitreous humor and on the inner layers of the retina. A "spectrum" of the crystalline lens thus obtained shows such a want of homogeneity of structure as would not be tolerated in any optical instrument of man's construction. Donders gives at page 200 of his "Treatise on the Anomalies of Refraction and Accommodation of the Eye" a very elegant spectrum of the crystalline lens of his right eye. In Fig. 48 is given a diagram of the spectrum of my right eye, showing the remains of the lines of union of the sectors ; the minor irregularities in the structure of the lens are not recorded in the diagram.

Fig. 48.

A SPECTRUM OF THE AUTHOR'S CRYSTALLINE LENS, SHOWING THE LINES OF UNION OF THE SECTORS.

§ 206. It is a legitimate inference that such optical imperfections should seriously impair visual acuteness, and this undoubtedly is the case. It is certain that our conventional standard of visual acuteness would be much higher than $^{20}/_{20}$ if it were not for these defects in the lens, and it is most probable that to a freedom from them is to be attributed that supernormal acuteness possessed by some persons—amounting often to $^{20}/_{10}$.

We have, however, become so accustomed to many of the manifestations of these defects, that we either ignore them or no longer regard them as abnormal.

§ 207. The most common of these appearances are the lines seen radiating from bright stars or distant points of light. We are perfectly well aware that the stars are in reality not rayed, but round, and yet so wide spread is this fault of vision that the conventional typical representation of a star has become to be a central bright spot with radiating bright streaks.

Fig. 49.

THE APPEARANCE OF A DISTANT POINT OF LIGHT TO THE AUTHOR'S
RIGHT EYE.

Only a very few persons are recorded as seeing the stars without these rays, from which we may infer the extreme rarity of a perfectly homogeneous crystalline lens. To my right eye a distant street lamp has the appearance shown in Fig. 49. On comparing this with the spectrum of the lens (Fig. 48) it will be seen how nearly they agree. There are eight rays to the star, and clear indications of eight sectors to the lens, and the directions of the rays coincide perfectly with the positions of the sectors.

. § 208. Another manifestation of this irregular astigmatism of the lens is *polyopia monocularis*, though it is seldom present as a normal condition sufficiently marked to attract attention. It can be experimentally demonstrated in the following manner: Take a small black dot on a white ground, or better still, a small white point on a black ground, such as can be ob-

tained by scraping the enamel from a visiting card on a piece
of black velvet, and arming the eye with a strong convex lens
(6 or 8 D) in order to render it myopic and avoid the contrac-
tion of the pupil associated with accommodation, bring it close
to one of the small white scales having a diameter of about
$^1/_3$ mm.   When the bright spot is brought very close to the
eye and passes within the point of distinct vision, it does not
become a broad even circle of diffusion, as it would do were
the crystalline lens perfectly homogenous, but breaks into a
series of grayish figures around a darker center.   Each one
of these figures is the image of the bright spot formed by its
corresponding lens sector.   Fig. 50, which is borrowed from
Helmholtz, shows the polyopia produced by the lens in the
right (*a*) and left (*b*) eye of that great scientist.

Fig. 50.
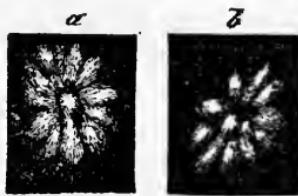
POLYOPIA MONOCULARIS (HELMHOLTZ).

In neither of my own eyes is the polyopia so marked as
that represented in these figures, from which I judge that my
lenses, though far from homogeneous, are yet more free from
inequalities of refraction than is usual.   Each one of these
images shows, likewise, fringes of colors, demonstrating the ex-
istence of a chromatic aberration as well.   I find this chromatic
phenomenon quite marked in my own eyes.

§ 209. Monocular polyopia is one of the most marked
features of *abnormal* irregular astigmatism of the lens, and is
most frequently found associated with incipient senile cataract.
The sclerosing process in the lens does not commonly affect
the different sectors equally, and the result is such a difference

in their refraction as leads to two or more images of the same object.   Sometimes the first indication of the formation of cataract is the appearance of two or more horns to the moon and multiple images of a distant street lamp.

§ 210. Dislocation of the lens, in addition to producing in certain instances a regular astigmatism, gives rise also to the irregular form, and a certain part, probably, of the normal irregular variety comes from a want of centering of the cornea and lens.   Injuries to the capsule and zonula which allow the lens to become irregularly curved will also have the same effect.

§ 211. Mauthner is of the opinion that the lenticular variety of irregular astigmatism is increased on accommodation for near objects.   He accounts for this by supposing that the capsule on relaxation of the zonula falls into folds which would, of course, mar the uniformity of the lens surface and lead to irregular refraction.   This wrinkling of the capsule he claims to have demonstrated as an actuality.

§ 212. If we make exclusion of the monochromatic oberration in the principal meridians demonstrated by Prof. Harkness, (§ 29) and that demonstrated by myself (§ 14) neither of which are correctible by cylinders, there is *no corneal astigmatism* of the irregular form which we can consider as *normal*.

And while, as we have seen, in the majority of instances lenticular astigmatism is congenital, with few exceptions irregular corneal astigmatism is acquired.   We meet, however, sometimes with a class of cases (and they are getting more numerous since we have now at command the proper means in keratoscopy for their easy detection) in which there are inequalities on the surface of the cornea that is still transparent, and where there is no history of a past inflammatory affection. In such cases there was, in all probability, an ulceration of the cornea during intra-uterine life.

It would appear from this that the formative processes in the cornea are much more regular and constant than those of the lens.

§ 213. The most common causes of irregular astigmatism in

the cornea are inflammations and injuries of its substance. The reparative processes following these pathological conditions are seldom so perfect as to leave the curvature or homogeneity unaltered ; and even a slight change in either of these particulars will be sufficient to affect in an appreciable manner the distinctness of the retinal inage. The amblyopia under such conditions does not depend solely on the opacity due to the effusion and organization of inflammatory material in the corneal substance, but results mainly from the distortion and blurring of the image caused by the irregular dispersion of the light rays, and this distortion and indistinctness of the image are quite as marked when the defect is unattended with any opacity, as, for example, in resorption ulcers with clear bottoms and edges. The condition of vision in these astigmatics is very well imitated by looking through a pane of very bad window glass which has irregular surfaces and inequalities in its substance.

§ 214. Diagnosis of Irregular Astigmatism.—Abnormal irregular astigmatism of the lens is readily determined in the following manner : The patient is caused to look with each eye separately at a small distant point of light, such as Donders used in his original method for determining regular astigmatism (§ 80), and if it does not appear single, but on the contrary, broken up into two or more spots, the diagnosis of irregular astigmatism is fixed; and if on further examination the cornea is found to be regular in curvature, its seat in the lens is placed beyond doubt.

This kind of astigmatism being most commonly met with in commencing cataract, the opacities of the lens associated with the change in the sectorial refraction can be seen by the oblique method of illumination or by simple illumination with the ophthalmoscopic mirror; in the latter case revealing themselves as dark or black lines or spots against the red background of the fundus.

§ 215. If there be a dislocation of the lens leaving its edge in the area of the pupil, the margin will be seen as a bright curved line on oblique illumination and as a curved black line

on direct observation of the fundus by the mirror alone. The bright line in the first instance is due to the reflection of the incident light from the edge of the lens, having, of course the color of the light used, being yellow if it is artificial and white if day light.

The black line in the other method comes from a total reflection of the light coming from the illuminated fundus along the edge of the lens, leaving a narrow unilluminated space in striking contrast to the otherwise brilliantly lighted background. It often happens that two images of the fundus visible at the same time can be obtained in the indirect method of ophthalmoscopic examination, one through the lens and one through the pupillary space which is free of the lens.

§ 216. Irregular corneal astigmatism is most easily diagnosed

Fig. 51.

KERATOSCOPIC APPEARANCE OF THE CORNEA IN A CENTRAL CORNEAL OPACITY.

by *keratoscopy*, and best by means of the disk of Placido described in § 156. Wecker's square (§ 158) can also be used, but it is much inferior to the concentric circles. In making the examination, the patient is seated with the back to a window and the figures are so held as to get a good reflection of them from the corneal surface. Any irregularity of curvature is then at once manifest in a distortion or unequal thickness of one or more of the circles, or a distorted form of the square. The shapes which they assume are sometimes quite fantastic. They will be best illustrated by the actual appearances in some cases selected from my case-book.

Figure 51 represents in *A* the appearance of Placido's disk,

and in *B* that of Wecker's square, as they were reflected from a cornea having an opacity near its center, the result of an ulcer. It will be observed that in this case there is also some regular astigmatism as shown by the drawing out of the circles and square from above inward downward and outward, a not unusual consequence of corneal ulceration.

Fig. 52.

KERATOSCOPIC APPEARANCE IN A SMALL CORNEAL INFILTRATION.

Figure 52 shows the reflection from a cornea in which there was a small circumscribed opacity resulting from an infiltration, the rest of the cornea being clear. V could not be brought to more than $4/60$ even after a correction of the myopia that was present.

Fig. 53.
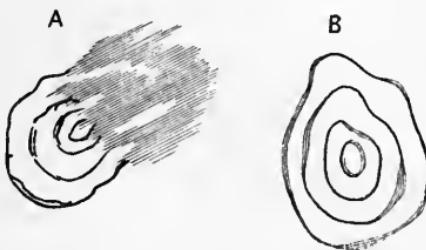
KERATOSCOPIC FIGURES IN A CASE OF CYSTOID CICATRIX.

§217. Should neither Placido's disk or Wecker's square be at command, the reflection of a window sash with its rectangular figures will show any irregularities that may be on the corneal surface. In this examination, of course, the patient must sit facing the window.

§ 218. The reflection figures in any one of these methods of keratoscopy are not the same from all parts of the cornea. The changes in form as the eye is moved in different directions, while the object is stationary, are quite kaleidoscopic in their character. Fig. 53 gives two forms of Placido's disk in a case of cystoid cicatrix situated at the upper inner sclero-corneal margin; one, *A*, from over the center of the pupil, showing the extreme flattening of the cornea in the direction of the scar, and the other, *B*, when the eye was turned 10° inward.

§ 219. Irregularities of the corneal surface, even where there are no opacities, can often be detected by direct inspection, and particularly when oblique illumination is used. The appearance of an ulcer with a clear bottom, for instance, is quite characteristic when the light is concentrated on it by a convex lens. The edge appears as a bright ring, the sides darker, on account of the reflection of the light out of the line of the observers' vision, and the center as a bright spot of light. The same appearances will be found also in a transparent circumscribed elevation of the surface. These ulcers and elevations, whether transparent or not, cast shadows on the anterior face of the iris. In the case of a transparent elevation, the shadow will have a bright center surrounded by a dark ring, while in the case of a depression there will be dark center with a brighter rim. The surface of the iris seen through these irregularities often presents a wavy appearance which changes as the point of view is changed.

§ 220. All these circumscribed alterations of curvature and opacities reveal themselves as dark spots on a red back-ground when the fundus is illuminated by the ophthalmoscopic mirror, the light from the bottom of the eye being either reflected or refracted by them in such a manner that little or none of it which should pass through them reaches the eye of the observer.

§ 221. A certain and often a considerable amount of irregular astigmatism of the cornea is manifest during the healing of the wound after *cataract extraction*, and particularly when it is complicated with inflammation either of the cornea itself or of the anterior portion of the uveal tract.

I extracted an opaque lens from the R eye of John O'N——— by means of Wecker's incision and an iridectomy. The operation was perfectly smooth and normal and no symptoms of irritation showing themselves he was allowed to leave the hospital at the end of the ninth day. Three days after, he presented himself again with a pronounced iritis. The wound which at the time of his discharge appeared well united showed signs of separation, and the lips were infiltrated and gray. I made measurements with the keratometer and found $115°\ r = 10$ mm. $10°\ r = 7^3/_4$ mm., a difference in refraction amounting to 13 D.

Fig. 54.

KERATOSCOPIC IMAGE AFTER CATARACT EXTRACTION.

A keratoscopic examination with the disk of Placido showed an amount of wrinkling of the cornea as exhibited in Fig. 54. The iritic inflammation was most persistent, and it was six weeks before there was any evidence of abatement. He was examined with the keratoscope and keratometer from time to time and there was no alteration observed until the inflammation began to subside. But even when there was no longer any inflammation going on, there was quite an irregularity in the course of the rings, and it was only after I had made a discision of the secondary cataract that the figure became regularly oval. This case shows quite conclusively that the contraction of the capsule especially when accompanied with inflammatory deposits is capable of drawing on the base of the cornea in such a manner as to wrinkle its surface.

§ 222. The operation of iridencleisis—now seldom performed—the operation for iridectomy, and, in fact, any traumatic injury to the cornea, or sclero-corneal margin is liable to be followed by a greater or less amount of irregularity in curvature which will mar the distinctness of the retinal image. In all cases of injury to the anterior portion of the eye where reduced visual acuteness follows, a keratoscopic examination should be made to determine how far irregularity in corneal curvature is the cause.

§ 223. Changes in the tension of the eye-ball—hypo-and

hypertony—often affect very markedly the character of the corneal curvature. The evenness of the corneal surface is not often affected by an increased tension, but where the tension is reduced some irregularity is seldom absent. Fig. 55 shows the reflection image of the disk from the left eye of Mrs. C., which was affected with ophthalmo-malacia in consequence of a chronic uveitis following a dislocation of the lens. The ball was much smaller than the other, and the tension was reduced to — 3.

Fig. 55.

KERATOSCOPIC IMAGE IN OPHTHALMO-MALACIA.

§ 224. The *symptoms* of irregular corneal astigmatism are essentially the same as those found in the same form of lenticular astigmatism. These are : a dazzling sensation which comes from the diffusion of light over the retina caused by the dispersion of the light rays by the semi-opaque spots, depressions, elevations and irregularities of curvature; diplopia or polyopia arising from the different images formed by the different parts of the same refractive surface ; distortion of the outlines of objects from an unequal refraction in the same plane—making a straight line, for example to appear crooked; amblyopia from the indistinctness of the principal retinal image due to the causes just stated ; and frequently an asthenopia, the expression of muscular strain from the necessity of holding work close to the eyes or of mental fatigue in attempting to obtain definite sensations from indistinct impressions.

§ 225. The regular form of astigmatism which is frequently

found associated with these corneal changes must be treated in the manner described in the chapter devoted to that subject.

§ 226.   In the *treatment* of the irregular form the indications are to cause the light to pass through the most regularly curved part of the cornea and to cut off that which passes through the portion which only serves, by its diffusion, to render the image indistinct.

§ 227. One way of accomplishing this is to place a small hole or a narrow slit, 1 to 2 mm. wide, in an opaque disk such as Donders used in his first investigation of regular astigmatism, opposite the most evenly refracting portion of the corneal surface.   By this means all rays are cut off except those going through the part corresponding to the slit, and this then represents practically the refracting surface of the cornea. We can then proceed to examine this part of the refracting media of the eye in the same way as when the whole cornea is exposed, and determine the character and degree of its ametropia should any be present.

In some cases we find a very considerable augmentation of visual power, particularly for near objects by this means, but it is not so applicable to distant vision on account of the narrowing of the visual field.   And as a matter of my experience, the improvement, after the correction of whatever ametropia (including regular astigmatism) that may be found on a careful examination, that is produced by the addition of the stenopaic apparatus, is not sufficient to justify its employment except in rare cases.

§ 228. Another method is to remove the pupil, either by making an iridectomy or by the operation of iridencleisis, to such a position that it shall lie behind the most regularly curved portion of the cornea.

It frequently happens that there is a central opacity of the cornea which covers the pupillary area, and we have a choice of several positions at which to make the artificial pupil.   We should, of course, under such circumstances, choose that place where there is least irregularity of corneal curvature.   This we are enabled to do by means of the concentric circles or the

square.   We get reflections successively from the various parts of the surface until one part is found which offers least irregularity of outline and make the pupil to lie under that.   Such iridectomies—as should all iridectomies made for visual purposes—must be small.

§ 229.   KERATOCONUS.   The cornea, as a result of inflammatory changes in its substance may become very much distended in all directions and assume somewhat the shape of a globe (kerato-globus); or it may become bulged out at a certain locality (keratectasia); or it may assume the form approximating that of a cone (keratoconus).   All of these conditions will give rise, of course, to the phenomena of irregular astigmatism.   These consequences of inflammation are most commonly associated with opacities which still further mar the distinctness of the retinal images.

But there is one form of keratoconus which is developed in a clear cornea without any changes in the transparency of the tissue.   It was known among the earlier writers as "staphyloma pellucida."   There are some reported cases in which the change was probably congenital, but for the most part it is one of gradual development.

§ 230.   The precise nature of these changes and the pathology lying at the root of them have not been definitely settled, but there is scarcely any doubt that the tissue of the cornea is thinned at its apex and thickened at its base, and that it is not the result of an inflammation of the substance of the cornea— at least in the ordinary acceptance of the term inflammation. It usually commences about the fifteenth year and stops about the twenty-fifth, though it is not always progressive during that period.   It occurs with much greater frequency in women than in men.

§ 231. The first of the more modern writers to describe this condition was Scarpa, in his treatise published in 1802.   As a matter of historic interest I quote the case recorded by him.[1]

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[1] From the French edition translated from the Italian by Fournier-Pescay and Bégin.   Paris.   1821.   Tome 2.   P. 214.

"Not long ago it happened to me to observe a peculiar affection of the cornea, which I am not able to classify unless it be under the heading of staphyloma. In a woman 35 years of age, with naturally prominent eyes, the centers of the two corneæ were protruded without any apparent cause, in such a manner that this membrane no longer formed the segment of a regular sphere affixed by its base to the sclerotic, but assumed exactly the shape of a pointed cone. Viewed laterally the cornea had the appearance of a small transparent funnel applied by its base to the sclerotic. During some movements of the globe the point of the cone seemed somewhat less transparent than the base; in other movements this effect was not apparent. Yet at the places where this transparency was least, there was sufficient to oppose a remarkable obstacle to vision. On placing the eye directly in front of a window the apex of the cone reflected the light to such an extent that it appeared as a brilliant point; and as this occurred directly in front of the pupil, already contracted, the woman could only see distinctly in a subdued light which allowed the pupil to dilate sufficiently; when the light was strong she could see only a little, and confusedly."

It appears, however that Beer[1] had noticed some condition similar to this, for he says.

"There is a kind of staphyloma worthy of remark, which I have seen in more than one case of hydrophthalmia. The cornea in such cases is inconceivably distended, but it does not lose its transparency. The patients, notwithstanding the transparency of the cornea, saw little or none at all."

The conical character of such a cornea seems to have escaped his notice.

Among the older writers mention is made of the "pellucid cornea" by St. Yves (1722), Manchart (1748), Taylor (1750), who gave it the name "Ochlodes;" Himly (1819), who gave it the name of "Keratosis," and v. Ammon (1831) who called it "Keratoconus."[2]

The first thorough examination of the optical phenomena presented by the conical cornea, I find in Wardrop.[1]

The examination was made by Dr. (afterwards Sir David) Brewster, and I add his account of it in full as given in a letter to Mr. Wardrop, (p. 119-20). It is interesting among other things from the fact that it is, perhaps, the first attempt at the employment of keratoscopy in the diagnosis of abnormal curvature of the cornea.

"When you first mentioned to me the case of Miss ——, I was much surprised at the number of images which she observed round luminous objects. As this multiplication of images could arise only from some irregularity in the cornea or crystalline lens which gave their surface the form of a polyhedron, it was completely inexplicable from the shape of the cornea itself which your drawing represented (Fig. 56) as a regular surface, resembling very much that of a hyperboloid; for the only indistinctness occasioned by a cornea of this kind would arise from the concentration of the rays before they fell upon the retina.

When I had the pleasure of examining the eye itself, the difficulty of explanation was in no respect diminished. In every aspect in which the cornea could be viewed its section appeared to be a regular curve, increasing in curvature toward the vertex; a form which could produce no derangement in the refraction of the incident rays.

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[1] Prakt. Beobactungen ü. d. grauen Starr u. d Krankheit. d. Hornhaut. Wien. 1791

[2] Essays on the Morbid Anatomy of the Human Eye. Edinburgh. 1808.

As the disease was evidently seated in the cornea, which projected to an unnatur-
distance, it did not seem probable that there was any defect in the structure of the
crystalline lens.   I was therefore led to believe that the broken and indistinct images
which appeared to encircle luminous objects arose from some eminences in the cornea
which could not be detected by a lateral view of the eye, but which might be rendered
visible by the changes which they produced upon the image of a luminous object
that was made to traverse the surface of the cornea.   I, therefore, held a candle at
the distance of fifteen inches from the cornea, and keeping my eye in the direction of
the reflected rays, I observed the variations in the size and form of the image of the
candle.   The reflected image regularly decreased when it passed over the most con-
vex parts of the cornea ; but when it came to the part nearest the nose, it alternately
expanded and contracted and suffered such derangements as to indicate the presence
of a number of spherical eminences and depressions which sufficiently accounted for
the broken and multiple images of luminous objects."

Fig. 56.

A LATERAL VIEW OF A CONICAL CORNEA.   (WARDROP).

§ 232. The *diagnosis* of this condition is oftentimes a matter
of no difficulty, a simple inspection of the cornea in profile be-
ing sufficient to show the conicity quite plainly.   There is also
very frequently a bright reflex at the corneal apex as if a
tear had fastened itself there.   This, however, is in the
higher forms of the anomaly.   In the lower developments these
gross changes are not so apparent, and we must then resort to
other methods of examination, when keratoconus is suspected.

§ 233. For this purpose there is nothing better than a careful
and systematic examination by means of the keratoscope.   In
this way we get, through the changes in the form of the reflec-
tion figures at various localities, a very good idea of the general
form of the surface, as well as of the changes at particular por-
tions.   The figures are small at the apex and generally in-
crease in size as they approach the periphery.   It occasionally
happens that this increase is very regular, showing how nearly
perfect the cone is.   The following is an illustration of this :

CASE I. It is the case of Mrs. R. a part of whose history relating to regular astigmatism was given in Chap. XI, § 175. She reports that up to her sixteenth year she saw well. At that time her vision began to fail and gradually got worse till her nineteenth year, since which time it has remained about as it is now. On October 6, 1884, the time of my first examination, V $=^{2.5}/_{60}$. With — 6s she saw with either eye No. 60 at 4 meters ; no other spherical glasses giving further improvement. As is my habit, I then made an ophthalmoscopic examination before trying cylindrical glasses, since I often obtain thereby important indications as to the directions of the meridians, the form of the astigmatism, etc., which will materially shorten the examination and add to its accuracy.

Fig. 57.
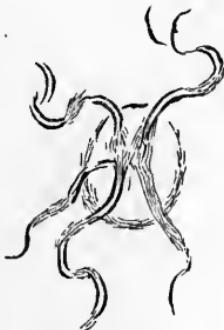
DISK AND LARGE RETINAL VESSELS AS SEEN IN KERATOCONUS, WHEN EXAMINED BY THE DIRECT OPHTHALMOSCOPIC METHOD.

I at once found that I had to do with a case of keratoconus. Even in the inverted image it was not possible to see all parts of the disk clearly at once, and when the auxiliary lens was moved from side to side there was that paralactic movement of the vessels which is characteristic of keratoconus when this method of examination is used. When the light from a plane mirror was thrown into the eyes from a distance, as in examination by skiascopy, the peculiar unstable shadow crescent of conical cornea was beautifully shown. Examination by the direct method was in the highest degree unsatisfactory. At no time and with no lens was it possible to get more than a small portion of two or three vessels in focus at once, and the slightest movement of the eye of the patient or of the ophthalmoscopic mirror would throw these out of view and bring others forward. Some idea of the peculiar distortion of the vessels may be obtained from Fig. 57, which represents diagrammatically the disk of the R eye and its immediate neighborhood as seen with a + 4 behind the ophthalmoscopic mirror. The black lines represent the parts of the vessels seen with distinctness, the shaded portions the parts that were out of focus.

The radius of curvature of the cornea was measured by the keratometer in various meridians and in different parts of the same meridian.   The most nearly regular portion was found, not directly in the line of vision, but about 5° outward in each eye. At this place in L 180° $r = 5^{mm}$ ($39^1/_2$ D), 90° $r = 6^{mm}$ (34 D) ; in R 180° $r = 6^3/_4{}^{mm}$ (30 D), 90° $r = 5^1/_2{}^{mm}$ (36 D).   But even in these meridians there was a great change in the figures as soon as the place of measurement was removed a few degrees from this point.   The shape of the bands became very much distorted and it was impossible to take accurate measurements.   It was apparent, however, that the corneal surface became flatter as it approached the periphery.   This distortion began much sooner on the outer side in both eyes.   I measured in L the meridian

Fig. 58.
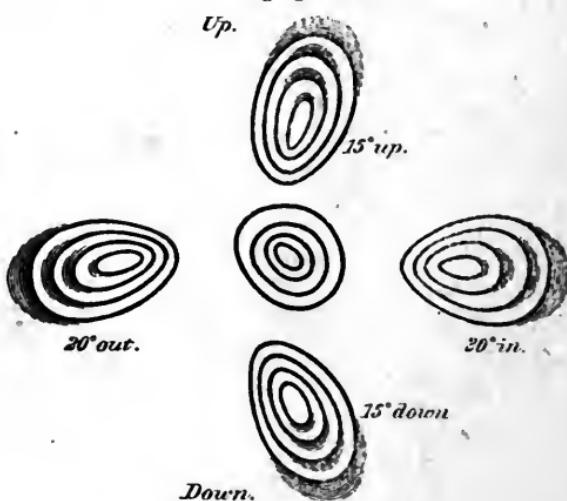
KERATOSCOPIC IMAGES AT VARIOUS PARTS OF A CONICAL CORNEA.

at 180°, 20° inward and outward from the apex, and found r inward $= 7.5^{mm}$, outward 8^{mm}.   The distortion was also greater in the upper than in the lower portion of the cornea.   An examination with the concentric rings revealed an approach to regularity of curve which I do not believe is commonly met with in keratoconus. Fig. 58 shows the form of Placido's disk at the center, and at 20° upward. downward, inward and outward.   It will be seen how pear-shaped it becomes when reflected from the sides of the cone, indicating a gradual flattening towards the base.   The similarity in the shape of the four lateral reflections shows an approximation to uniformity in curvature at corresponding distances from the apex, resembling, somewhat, an ellipsoid of revolution, though we know from measurements that it is compressed laterally.   The apex itself, however, offers a very considerable amount of irregularity.

§ 234. Such regularity is quite exceptional, the majority of cases being more like the following :

CASE II. Miss L., æt. 19, says she had fairly good vision up to three years ago. Since that time it has gradually deteriorated until now she can barely count fingers at 3 meters. In Fig. 59 are shown enlarged views of the reflections of Placido's disk from the corneal apex of both eyes. The image became larger as it approached the periphery in all directions, showing an increased radius of curvature. Not having a keratometer at command at that time no measurements of radii were taken. The reflection figures of Wecker's square are shown below those of the circles in Fig. 59.

Fig. 59.
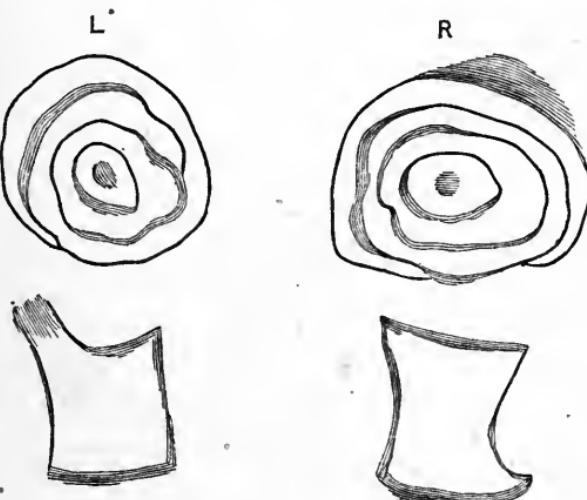
KERATOSCOPIC IMAGES IN CONICAL CORNEA.

The irregularity of these is in striking contrast to the uniformity of the disk in th case of keratoconus just described. No glasses improved distant vision. With the unaided eye she could read, L, No. 5 of Wecker's scale, and R, No. 7, at 6 to 8 inches. When a diaphragm, having a hole 1 mm. in diameter was placed before the eye, she could read, L No. 1, R, No. 4, of the same scale at the same distance. Distant vision was not materially improved through this stenopaïc hole.

§ 235. There are other methods of diagnosis, which, though less accurate than keratoscopy, are nevertheless of value and bring out very characteristic appearances.

Among these the *ophthalmoscope* ranks first. When light is thrown into the eye with the mirror, as in examination by skiascopy, the center of the pupillary area appears bright, with an ill-defined dark ring or crescent separating it from the periphery,which is also bright, but usually less so than the center. This dark ring or cresent is not fixed, but very vacillat-

ing, moving with each change in the position of the eye or mirror. In the majority of cases, when the mirror is at a proper distance from the eye, an inverted image of a part of the fundus can be seen without the aid of an auxiliary lens, through the central part of the cornea. This image, which may often not be more than a portion of a single retinal vessel, moves in the same direction as the mirror and nearly always changes its shape with a change of its position.

All these phenomena are due to the optical properties of the corneal cone. The central part, on account of its excessive curvature, is strongly myopic and concentrates the rays coming from the illuminated fundus at its far point some inches in front of the eye, making a brilliantly illuminated area, together with an inverted image of the objects lying in that part of the fundus. Those rays which fall on the sides of the cone at a certain angle suffer total reflection, rendering this portion of the ophthalmoscopic field much darker in comparison with the central area. Those rays falling on the more peripheral and flatter parts of the cornea pass through, but, being more scattered than the central ones fewer of them reach the eye of the observer; hence this circumferential portion of the field is less brilliant than the central.

Examination by the direct ophthalmoscopic method furnishes equally characteristic phenomena. It is impossible to get a clear and distinct view of all the parts in the entire ophthalmoscopic field at once. A vessel, for instance, will appear with sharply-defined outlines at a certain part and then suddenly become blurred and thrown out of its course. The outline of the disk is not distinct in all its parts, and the apparent curvings and twistings of vessels are often quite fantastic; the whole presenting an appearance which might easily be mistaken by a novice for a pathological condition.

Fig. 57 is intended to give some idea of how the fundus appears under these conditions. The most marked phenomenon, however, in connection with these appearances is its changeableness. The slightest movement of the head or mirror throws some parts clearly in view out of the focus, and brings others, hitherto obscure, forwards.

There are also appearances peculiar to this form of irregular refraction when the examination is made by the indirect method.   Even when the inverted image is clear and distinct throughout, there are differences in the paralactic movements of the different parts which make the diagnosis certain.   If the refracting media are symmetrical in their refraction as a whole, when the auxiliary lens is moved in any direction perpendicular to the line of vision, the image moves with it as a whole, because all parts of the image are formed on the same plane; but if there is such irregularity in refraction that parts of the image will be formed in several different planes, movements of the lens will be accompanied by unequal movements of these separate parts of the image, some moving much more rapidly than others.   And in keratoconus where the different parts of the cornea have different refractions these parallax motions are sometimes very striking.

§ 236.   The subjective symptoms of keratoconus do not differ in any essential particulars from those of the other forms of irregular astigmatism.   Vision is always impaired, and distant V is much worse in comparison than near, and occasionally there is a complaint of polyopia monocularis.   Metamorphopsia or a distortion of images is also a not infrequent accompaniment.

§ 237.   *Treatment of Keratoconus.*—Keratoconus may be treated optically or by operation.

The strictly *optical* treatment has not until quite recently found favor with the profession.

Donders gives in his treatise no suggestion of glasses, and the only mention Mauthner makes of them is to express an opinion of the worthlessness in general of cylindrical lenses in this class of cases.

The *stenopaïc slit or hole* was the only means, other than operative, which it was thought worth while to employ for the improvement of vision.   This apparatus, by excluding all the rays except those going through a limited portion of the cornea cuts off a large amount of diffused light that is very destructive to the distinctness of the retinal image. These patients

almost universally, by instinct, make use of such an apparatus by narrowing the palpebral aperture. It is not always a matter of indifference over what part of the cornea the slit or hole lies, and it should be placed successively in different positions until that one is found in which vision is clearest; and it not infrequently happens that the addition of a spherical or cylindrical glass contributes still further to the distinctness of the image. The advantage of the stenopaic apparatus is confined almost entirely to near vision, and this is often very markedly improved. It is seldom useful for distant vision on account of the diminished illumination and the restriction of the visual field.

§ 238. But a great improvement in vision can be effected in a large number of cases by means of spherical or cylindrical glasses alone. There are few cases in which there is not a greater or less amount of regular astigmatism; and in some instances the benefit derived from a correction of this by cylindrical glasses is very great as shown in the case reported in § 175. The chief obstacle that has hitherto lain in the way of a more general employment of cylinders in such cases is doubtless the great difficulty usually experienced in unravelling the optical complexities inherent in the condition. Few surgeons have the time or patience to work out the problem with lenses and test objects alone where there is so high a degree of amblyopia. The ophthalmoscope offers little or no assistance in the task. It is here that keratometry and keratoscopy find one of the fields for their most satisfactory application. A simple inspection of the corneal reflection of the concentric rings is sufficient to give us the direction of the principal meridians, and if a keratometer is at hand, it is easy to find the difference in the refraction of these two meridians, expressed in dioptries, data which will render the further determination of the refraction of the eye comparatively easy.

§ 239. We have seen in § 186 *et seq* how cylindrical lenses fail to give complete correction to the corneal ellipsoid in regular astigmatism. It is apparent that they will be much less effective in correcting the paraboloid curve in keratoconus.

Such a surface can be neutralized only by a lens which approaches to a hyperbola in form.

§ 240. Rählmann, of Dorpat, was the first to use such lenses for the correction of conical cornea. At the meeting of the HeidelbergCongress of Ophthalmologists in 1879 he exhibited these lenses for the first time with the report of a case in which they had been successfully applied. The best V to be obtained by concave spherical and cylindrical glasses was $\frac{1}{10}$; with the hyperbolic lenses it advanced to $\frac{1}{2}$. Since then the subject has been further worked up by Rählmann himself, and several others, and the advantage of such lenses now seems established beyond question. All cases, however, are not benefited in the same degree, and some are not benefited at all. Up to this time their selection has been somewhat empirical, but since we have now a ready means for measuring the corneal curvature at its various parts, it seems possible that we may be able to determine the proper correcting lens with greater scientific exactness. The principal difficulty at first will be in having the glasses manufactured with the curve found necessary by the measurements, but if further experience demonstrates the promising usefulness of these lenses, optical art will, as it has always done, keep abreast with the demands made upon it.

Rählmann employs two series of hyperbolic lenses, which differ from each other in the length of the hyperbola axis. Series "A" has an axis of $\frac{1}{4}$ mm., series "B" an axis of 2 mm. The different members of each series are numbered according to the size of the assymptote angle. The larger this angle is the less the surface is curved, and the weaker the refracting power and *vice versa*. When the assymptote cone with a base of 30 mm. has a height of 1 mm. (the height depending on the size of the assymptote angle) it is No, 1 ; when it has a length of 2 mm., with a smaller assymptole angle, it is No. 2, and so on. The members in series "A" having a shorter axis to begin with are much stronger than those in series "B" and are more pointed, and consequently better adapted to the sharper corneal cones. Angelucci has found great advan-

tage from a combination of these lenses with cylinders in the correction of keratoconus.

§ 241. The *operative* treatment of keratoconus is of two kinds. One has for its object a change in the shape or position of the pupil; the other aims at a reduction in the curvature of the cone.

The operations on the pupil consist either in making an iridectomy (as small as possible) under the most regularly curved portion of the cornea or in removing the pupil by the operation of iridesis, as first done by Critchett, to the same desirable locality. This last operation leaves a slit-like pupil which offers the same advantages as a stenopaic apparatus. Bowman modified it by making the operation at opposite sides (double iridesis) causing the slit to extend all the way across the width of the iris. This is, however, a questionable improvement, and is probably not now performed by any one.

§ 242. It has also been thought by some that the performance of an iridectomy has stopped the progress of the corneal change during the period of its development. Others have noted an arrest of the process under the local use of atropine and of eserine.

§ 243. The treatment of the cone itself is addressed to a flattening of its apex and a reduction of its curvature. This is accomplished by a removal of the top of the cone by operation and the subsequent cicatrization of the wound. The cicatrization is aided sometimes by caustic applications. Special forms of trephine have been been devised by Bowman and Wecker to remove a small circular piece from the corneal apex. For the manner of performing these operations and a description of the special instruments used, the reader must be referred to the chapter on operations in the text-books of ophthalmology and the various articles on the subject whose titles are to be found in the appended bibliography. It must be said, however, in regard to all these operative procedures that the opening of the anterior chamber to such an extent, is, at least, a hazardous undertaking, involving as it does the integrity of both cornea and lens, and should never be undertaken until all optical means have failed.

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NOTE TO SECTION 196—Pp. 170–171.

Since the text of this work has been put in type, I have made a series of experiments which render intelligible some of the peculiar optical properties of spherical lenses when held obliquely to incident pencils of light. I used in these investigations a Snellen's phakometer, where the sources of the pencils are several small holes near the periphery and at the center of a disk 2.5 cm. in diameter, through which light, whose rays have been rendered parallel, passes. The lens to be examined is placed in the path of these rays, and turned on its vertical axis and its focus found by means of the image of these holes on a movable screen, for various degrees of inclination. The investigations of Pickering and Williams have already shown the cylindrical action thus acquired by a spherical lens (see Table II, page 34), but their measurements only give the results of the rays passing through the central portion of the lens. My experiments were directed to finding if there was also a difference in the focus of the rays refracted by the edge of the lens nearest the object and that corresponding to the edge farthest removed from it. This I found to be the case and the amount of the difference was very decided for even medium angles of inclination. *I found that the cylindrical focal plane of a spherical lens placed obliquely to the incident pencils, lies obliquely to the optical axis, and in a sense contrary to the inclination of the lens.*

In Table VII are given the foci of a spherical lens of +1D for every 5° of inclination up to 45°, beyond which accurate results cannot be obtained on account of the general diffusion of light. In the second column is given the increase of the general spherical refraction, and in the other two columns the foci expressed in dioptries and decimals, at the side of the lens nearest the object (being the image of the hole on the opposite side) (F) and at the side farthest from it (F¹), the last, of course, being the image of the hole on the side of the lens nearest the object.

TABLE VII.

Degrees of Inclination.	Spherical Action.	F.	F.[1]
5°	0	Slight.	Slight.
10°	1.1	1.20	1.25
15°	1.2	1.30	1.5
20°	1.25	1.57	1.75
25	1.30	1.7	2.2
30	1.4	2.6	3.
40	1.6	3.6	5.1
45	1.75	5.2	6.8

I also experimented with other lenses, but will only add the results with a lens of $+8$ D., giving the focus at the center of the lens($F°$) in addition to the two at the periphery.

Degrees of Inclination.	Spherical Action.	F.	F°.	F[1].
10°	8.1	8.3	8.4	8.5
20°	8.2	8.7	9.	9.5
30°	8.5	10.	10.5	11.3
40°	9.	13.5	15.	17.

The superiority of obliquely placed spherical to the ordinary cylindrical lenses, in some cases of cataract extraction, is probably due to the correction of a difference in the refraction in the different parts of the same meridian, (generally the vertical, since the incision is always made above or below) by this lens whose refraction gradually increases from one extreme of a corresponding meridian to the other. In other words, it corrects a certain amount of irregular astigmatism resulting from the corneal wound.

Keratometric measurements, in these cases of cataract extraction, at various points on this meridian should enable us to measure exactly the amount and character of this form of irregular astigmatism and furnish data for correction of the defect, either by a lens inclined to a certain degree, or a lens ground in such a manner as to give the same optical effect.

I will also state, in this connection, that I found the same results as to character and degree when a cylinder was rotated on its axis. The cylindrical action as a whole was increased, thus confirming the views of Hay, and opposing those of Sous (see § 26, pp. 34 and 35), butt here was also the same inclination of the focal plane as was found in the obliquely placed spherical lens.

APPENDIX.

A STATISTICAL RECORD OF 806 ASTIGMATIC EYES.

I have collected in the following table the data furnished by 475 cases taken in the order in which they were recorded in my private case-book during the last five years, and which, I hope, may be of some value to the future student of astigmatism. These 475 persons had 806 astigmatic eyes, showing that in about 41% of the cases the anomaly was unilateral.

In 291 of the eyes affected with all degrees of astigmatism the visual acuteness was brought, by correction, up to the normal standard of $^4/_4$, being about 36%. This is a better showing than that alluded to on page 168 where in 2,000 cases there was normal vision in only 10%. This computation was made principally from Snellen's and Van Haaften's statistics, and the difference between the two is probably due to the fact that I have corrected the lower degrees more frequently than they. My clientéle is drawn largely from the clerical force in the various departments at the National Capital, where the work is of such a nature as to cause small errors in refraction to be felt, particularly by women.

To this latter fact is due the preponderence of women in my tables, there being 276 of them to 199 men.

In the higher forms it will be seen on a consultation of the tables a normal visual acuteness is rarely found. In 504 of the 806 eyes the principal meridians were vertical and horizontal.

The relative frequency of the various forms was as follows:

Simple myopic astigmatism - - - - -	294	or 37.%.
Compound myopic astigmatism - - - - -	162	or 20.%.
Simple hypermetropic astigmatism - - - -	210	or 26.%.
Compound hypermetropic astigmatism - - -	113	or 14.%.
Mixed astigmatism - - - - - - -	27	or 3.%.
Total - - - - - - - -	806	100.

A STATISTICAL RECORD OF 806 ASTIGMATIC EYES.

Number.	Name.	Sex and Age.	Correcting glasses and direction of axis of cylinder.	Vision after correction.
1	B. F. M.	M., 40.	L.$+^1/_{36}$ 90°. R.$+^1/_{36}$ 90°.	$^{20}/_{20}$ $^{20}/_{20}$
2	C. M.	M., 36.	L.$+$0.5 90°. R.$+$0.25 90°.	$^4/_4$ $^4/_4$
3	R. D. D.	M., 26.	L.$+^1/_{60}$ 90°. R.$+^1/_{60}$ 90°.	$^4/_5$ $^4/_5$
4	V. Mc.	F., 17.	L.$-^1/_{30}$ 180°. R.$-^1/_{30}$ 180°.	$^4/_6$ $^4/_6$
5	A. Mc.	F., 38.	L.$-^1/_{30}$ 180°. R.$-^1/_{30}$ 180°.	$^4/_6$ $^4/_6$
6	H. H. H.	F., 20.	L. Em. R.$-^1/_{40}$ 180°.	$^4/_6$
7	R. M.	M., 23.	L.$-$2.25 $\smile$ $-$0.75 90°. R.$-$2.25 $\smile$ $-$0.75 130°.	$^4/_4$ $^4/_4$
8	M. S. F.	F., 40.	L.$-^1/_{40}$ 180°. R.$-^1/_{40}$ 180°.	$^4/_4$ $^4/_4$
9	E. T. F.	F., 11.	L.$-^1/_{10}$ 180°. R.$-^1/_{10}$ 180°.	$^4/_9$ $^4/_9$
10	C. M.	M., 38.	L.$-^1/_{60}$ 10°. R.$-^1/_{60}$ 170°.	$^4/_4$ $^4/_4$
11	G. K.	M., 24.	L.$+$0.5 90°. R.$+$0.5 90°.	$^4/_4$ $^4/_4$
12	S. W. B.	M., 26.	L.$-$0.75 180°. R.$-$0.75 180°.	$^4/_4$ $^4/_4$
13	E. P.	F., 10.	L.$-$1 $\smile$ $-$0.25 180°. R.$-$2 $\smile$ $-$0.5 180°.	$^4/_5$ $^4/_5$
14	G. R.	F., 42.	L.$+$0.75 140°. R.$+$1.5 20°.	$^4/_6$ $^4/_6$
15	H. M.	F., 14.	L.$-$5 $\smile$ $-$1.5 20°. R.$-$5 $\smile$ $-$1.5 130°.	$^4/_6$ $^4/_6$
16	A. C.	F., 21.	L.$+$2.25 90° $\smile$ $-$3.5 180°. R.$+$3.5 90° $\smile$ $-$1.75 180°.	$^4/_6$ $^4/_6$
17	H. D. B.	M., 25.	L.$-$7 $\smile$ $-$1.75 15°. R.$-$7 $\smile$ $-$1.75 160°.	$^4/_6$ $^4/_6$
18	J. L. E.	M., 30.	L.$+$2.5 90°. R. Em.	$^4/_{12}$

Number.	Name.	Sex and Age.	Correcting glasses and direction of axis of cylinder.	Vision after correction.
19	J. W. H.	F., 49.	L.+0.5 70°. R.+0.5 120°.	4/4 4/4
20	L. G.	F., 12.	L.+0.75 ⊃ 0.5 90°. R.+0.75 ⊃ 0.5 90°.	4/4 4/4
21	W. L.	M., 14.	L.+6 ⊃ +1.5 180°. R.+6 ⊃ +1.5180°.	4/5 4/5
22	L. B. H.	M., 35.	L.—0.75 70°. R.—0.75 120°.	4/4 4/4
23	O. D. W.	M., 25.	L.+0.75 90°. R.+0.5 90°.	4/4 4/4
24	M. E.	F., 24.	L.+0.5 90°. R.+0.5 90°.	4/4 4/4
25	H. H. M.	M., 34.	L.+0.5 90°. R.+0.5 90°.	4/4 4/4
26	L. E. B.	M., 28.	L.—0.75 180°. R.—0.5 180°.	4/4 4/4
27	E. W. N.	F., 39.	L.—1 ⊃ —2 165°. R.+1 90°.	4/4 4/4
28	T. A.	F., 45.	L. Em. R.+0.25 180°.	4/5
29	H. A. H.	F., 54.	L.—1 180° ⊃ + 2.5 90°. R.+1.5 ⊃ +1 90°.	4/12 4/6
30	I. M. C.	F., 22.	L.+0.5 90°. R.+0.5 90°.	4/4 4/4
31	A. C.	F., 45.	L.—0.5 180°. R.—0.5 180°.	4/4 4/4
32	T. E. M.	M., 25.	L.—0.5 ⊃ —0.5 180°. R. Amblyopic.	4/4
33	A. M. M.	M., 34.	L.—0.5 180°. R.—0.5 180°.	4/4 4/4
34	B. D.	F., 20.	L. Amblyopic. R.—6 180°.	4/9
35	S. F. T.	M., 30.	L.—6 ⊃ —1 180°. R.—3 ⊃ —1 140°.	4/4 4/9
36	T. T.	M., 41.	L. Em. R.—1 180°.	4/5
37	M. G.	F., 18.	L.—8. R.—8 ⊃ —1 180°.	4/6 4/6

Number.	Name.	Sex and Age.	Correcting Glasses and direction of axis of cylinder.	Vision after correction.
38	C. E.	M., 46.	L. E. R.+0.75 180°.	$\frac{4}{9}$ $\frac{4}{9}$
39	A. C.	F., 64	L.+1.75. R.+0.75 9⸳°.	$\frac{4}{18}$ $\frac{4}{18}$
40	J. F. K.	M., 22.	L.—4 ⌣ —1.5 150°. R.—5 ⌣ —1.75 5°.	$\frac{4}{5}$ $\frac{4}{5}$
41	C. C. N.	F., 36.	L+2 ⌣ +2 90°. R.+0.75 S.	$\frac{4}{9}$ $\frac{4}{9}$
42	O. DeF.	M., 50.	L.+0.75. R.+1 ⌣ +0.5 180°.	$\frac{4}{5}$ $-\frac{4}{5}$
43	E. H.	F., 18.	L.—0.5 18°. R.—0.5 180.	$\frac{4}{4}$ $\frac{4}{4}$
44	J. P.	M., 33.	L.—4 ⌣ —1 180° R.—5.	$\frac{4}{4}$ $\frac{4}{4}$
45	J. N. P.	M., 46.	L.—0.5 90°. R. Em.	$\frac{4}{4}$ $\frac{4}{4}$
46	C. H. B.	M., 58.	L.—0.75 ⌣ —0.75 95°. R.—0.5 ⌣ —0.5 90°.	$\frac{4}{5}$ $\frac{4}{5}$
47	N. G. D.	F., 30.	L.—7 ⌣ —1 90°. R.—3.5.	$\frac{4}{18}$ $\frac{4}{5}$
48	E. H.	M., 50.	L.—3.5 ⌣ —0.75 180°. R.—3.5.	$\frac{4}{12}$ $\frac{4}{12}$
49	S. H. P.	M., 35.	L.—8 ⌣ —1 180°. R.—8 ⌣ —1.5 180°.	$\frac{4}{18}$ $\frac{4}{16}$
50	M. W.	F., 17.	L.—2 ⌣ —0.5 180°. R.—2 ⌣ —0.5 70°.	$\frac{4}{4}$ $\frac{4}{4}$
51	E. W.	F., 50.	L.—0.5 180°. R.—0.5 180°.	$\frac{4}{4}$ $\frac{4}{4}$
52	C. H. W.	M., 21.	L.—0.5. R.+1.5 ⌣ +1 90°.	$\frac{4}{4}$ $\frac{4}{18}$
53	J. A. W.	F., 38.	L.—0.75 180°. R.—0.5 180°.	$\frac{4}{4}$ $\frac{4}{4}$
54	J. T. W.	M., 40.	L.—1.5 20° ⌣ +2.75 110°. R. Em.	$\frac{4}{6}$ $\frac{4}{6}$
55	H. J. H.	F., 36.	L.+0.5 180°. R.+1.75.	$\frac{4}{4}$ $\frac{4}{18}$
56	E. D.	F., 38.	L.+0.75 90° ⌣ —0.75 180°. R.+0.75 90° ⌣ —0.75 180°.	$\frac{4}{4}$ $\frac{4}{4}$

Num-ber.	Name.	Sex and Age.	Correcting Glasses and direction of axis of cylinder.	Vision after correction.
57	W. H.	M., 31.	L.—7 ◠ —2 180°. R.—7 ◠ —1 180°.	4/6 4/6
58	C. H.	F., 28.	L.+1 180°. R +1.25 10°.	4/4 4/4
59	L. G. B.	F., 48.	L.—3 180° ◠ +0.75 90°. R.—1 180°.	4/5 4/5
60	C. E.	F., 40.	L.+0.5 90° ◠ —0.5 180. R —0.75 180°.	4/5 4/5
61	M. D.	F., 25.	L.—1 5 60°. R.—1.5 120°.	4/5 4/5
62	M. B.	F., 14.	L.+0.5 140°. R.+0.5 140°.	4/4 4/4
63	R. A. M.	M., 28.	L.+1.25 ◠ +1 180°. R.+2.5.	4/5 4/5
64	H. W. B.	M., 31.	L.+3.5 ◠ +1 90°. R. Em.	4/6 4/6
65	C. S. E.	M., 36.	L.—2 180° ◠ +0.75 90°. R.—2.5.	4/5 4/5
66	E. B. J.	F., 40.	L.+0.5 90° ◠ —0.5 180°. R.+0.5 90°.	4/4 4/4
67	G. H.	M., 11.	L.—0.75 180°. R.—0.5 180°.	4/4 4/4
68	J. W. H.	F., 35.	L.—0.5 90°. R.—0.5 90°.	4/5 4/5
69	C. S. B.	M., 36.	L.+0.75 90°. R.+0.5 90°.	4/4 4/4
70	M. C.	F., 25.	L.—3 ◠ —2.5 80°. R.—7.	4/9 4/9
71	W. H. L.	M., 22.	L.—0.75 90°. R.—0.75 90°.	4/4 4/4
72	T. J.	M., 49.	L.—0.5 90°. R. Em.	4/4 4/4
73	B. J.	F., 48.	L.+1.5 ◠ +0.5 180°. R.+1.5.	4/5 4/5
74	C. A. B.	M., 20.	L.—9 ◠ —2 180°. R.—9 ◠ —2 180°.	4/12 4/12
75	J. A. C.	M., 43.	L.+0.75 180°. R, Em.	4/4

Num-ber.	Name.	Sex and Age.	Correcting Glasses and direction of axis of cylinder.	Vision after correction.
76	D. M.	M., 39.	L.—0.75 125°. R. Em.	$\frac{4}{4}$
77	J. N.	F., 13.	L.+1.25 ⊃ +0.5 90°. R+1.25 ⊃ +0.5 90°.	$\frac{4}{5}$ $\frac{4}{5}$
78	M. P. B.	M., 22.	L.+4 ⊂ +3 5 90°. R.+6 90°.	$\frac{4}{18}$ $\frac{4}{18}$
79	J. H.	M., 64.	L.—2.5 ⊃ —1 90°. R.—2.5 ⊃ —1 90°.	$\frac{4}{4}$ $\frac{4}{4}$
80	M. J. S.	F., 35.	L.+2.5 ⊃ + 1.25 60°. R.+2 5 ⊃ + 1.25 115°.	$\frac{4}{5}$ $\frac{4}{9}$
81	A. N.	F., 18.	L.—4.5 ⊃ —1.5 180°. R.—4.5.	$\frac{4}{4}$ $\frac{4}{4}$
82	A. H.	M., 42.	L.+0.5 90°. R.+0.5 130°.	$\frac{4}{4}$ $\frac{4}{4}$
83	M. A.	F., 19.	L.—$\frac{1}{30}$ 180°. R. Em.	$\frac{20}{40}$
84	M. H.	F., 13.	L.+$\frac{1}{36}$ 90°. R.+$\frac{1}{28}$ 90°.	$\frac{20}{40}$ $\frac{20}{40}$
85	S. B.	F., 18.	L.+$\frac{1}{10}$ ⊃ +$\frac{1}{30}$ 90°. R.+$\frac{1}{10}$ ⊃ +$\frac{1}{30}$ 90°.	$\frac{20}{30}$ $\frac{20}{30}$
86	M. K.	F., 14.	L.—$\frac{1}{6}$ 140°. R.—$\frac{1}{6}$ 20°.	$\frac{20}{30}$ $\frac{20}{30}$
87	W. N.	M., 31.	L.+$\frac{1}{16}$ ⊃ +$\frac{1}{30}$ 90°. R.+$\frac{1}{16}$ ⊃ +$\frac{1}{30}$ 90°.	$\frac{20}{20}$ $\frac{20}{20}$
88	M. W.	F., 10.	L. (Staphyloma of cornea). R.— $\frac{1}{30}$ 80° ⊃ +$\frac{1}{30}$ 170°.	$\frac{20}{20}$
89	A. B.	F., 18.	L.+$\frac{1}{36}$ 60°. R.—$\frac{1}{32}$ 180°.	$\frac{20}{20}$ $\frac{20}{20}$
90	E. S. S.	M., 50.	L.—$\frac{1}{24}$. R.—$\frac{1}{42}$ 180°.	$\frac{20}{20}$ $\frac{20}{20}$.
91	T. M. S.	M. 45.	L.+$\frac{1}{36}$ 180°. R.+$\frac{1}{36}$ 150°.	$\frac{20}{20}$ $\frac{20}{20}$
92	C. M.	F., 30.	L.—$\frac{1}{40}$ 180°. R.—$\frac{1}{40}$ 180°.	$\frac{20}{20}$ $\frac{20}{20}$
93	A. P.	F., 33.	L.+$\frac{1}{14}$ 70°. R.+$\frac{1}{14}$ 110°.	$\frac{20}{20}$ $\frac{20}{20}$
94	H. H. B.	M., 29.	L.—$\frac{1}{30}$ ⊃ — $\frac{1}{60}$ 180°. R.—$\frac{1}{30}$ ⊃ —$\frac{1}{60}$ 180°.	$\frac{4}{5}$ $\frac{4}{5}$

Number.	Name.	Sex and Age.	Correcting Glasses and direction of axis of cylinder.	Vision after correction.
95	M. N.	F., 13.	L.—8 ⌒ —2.25 180°. R.—8 ⌒ —2.25 180°.	$\frac{4}{12}$ $\frac{4}{12}$
96	M. G.	F., 20.	L.+$\frac{1}{14}$ 90°. R.+$\frac{1}{42}$ 90°.	$\frac{4}{4}$ $\frac{4}{4}$
97	A. H. M.	M., 19.	L.+$\frac{1}{16}$ 90°. R.+$\frac{1}{16}$ 90°.	$\frac{4}{9}$ $\frac{4}{9}$
98	M. W.	F., 22.	L.—2 180°. R.—2 180°.	$\frac{4}{6}$ $\frac{4}{6}$
99	A. H.	F., 45.	L.+$\frac{1}{16}$ ⌒ +$\frac{1}{16}$ 90°. R.+$\frac{1}{14}$.	$\frac{4}{24}$ $\frac{4}{26}$
100	J. F.	F., 42.	L.—2.25 180°. R.—2.25 180°.	$\frac{4}{5}$ $\frac{4}{5}$
101	E. C. M.	M., 39.	L.—1.25 180°. R.—1.25 180°.	$\frac{4}{5}$ $\frac{4}{5}$
102	M. S. T.	F., 20.	L.—$\frac{1}{42}$ 180°. R.—$\frac{1}{42}$ 180°.	$\frac{4}{4}$ $\frac{4}{4}$
103	J. L.	F., 20.	L.—$\frac{1}{36}$ ⌒ —$\frac{1}{42}$ 180°. R.—$\frac{1}{28}$ ⌒ —$\frac{1}{42}$ 180°.	$\frac{4}{5}$ $\frac{4}{5}$
104	O. J.	M., 40.	L.+$\frac{1}{42}$ 90°. R.+$\frac{1}{42}$ 90°.	$\frac{4}{4}$ $\frac{4}{4}$
105	G. L. P.	F., 30.	L.—$\frac{1}{20}$ 180°. R.—$\frac{1}{20}$ 180°.	$\frac{4}{5}$ $\frac{4}{5}$
106	E. M.	F., 40.	L.+$\frac{1}{42}$ 90°. R.+$\frac{1}{42}$ 90°.	$\frac{4}{6}$ $\frac{4}{6}$
107	B. H.	F., 15.	L.—0.5 180°. R.—0.5 180°.	$\frac{4}{4}$ $\frac{4}{4}$
108	J. C. D.	M., 45.	L. Em. R.—0.75 180°.	$\frac{4}{4}$ $\frac{4}{4}$
109	M. W. L.	F., 18.	L.—0.5 180°. R.—0.5 180°.	$\frac{4}{5}$ $\frac{4}{5}$
110	F. A. P.	F., 19.	L.—1 ⌒ —0.75 180°. R.—1.75 ⌒ —0.75 180°.	$\frac{4}{9}$ $\frac{4}{9}$
111	L. P. S.	M., 20.	L.+0.5 90°. R.—0.75 180.	$\frac{4}{4}$ $\frac{4}{4}$
112	J. B. D.	M., 32.	L.—0.25 180°. R.—0.25 180°.	$\frac{4}{4}$ $\frac{4}{4}$
113	G. W. P.	F., 31.	L.—0.5 180°. R.—0.5 180°.	$\frac{4}{4}$ $\frac{4}{4}$

Num-ber.	Name.	Sex and Age.	Correcting Glasses and direction of axis of cylinder.	Vision after correction.
114	H. H. Y.	F., 36.	L.+ 1.50 160 ◯ —0.75 20°. R.—0.75 90°.	$\frac{4}{5}$ $\frac{4}{4}$
115	C. V. R.	M., 40.	L.—0.5 15°. R.—0.5 90°.	$\frac{4}{4}$ $\frac{4}{4}$
116	L. B.	F., 18.	L.—0.5 90°. R.—0.25 90°.	$\frac{4}{4}$ $\frac{4}{4}$
117	J. G. W.	M., 50.	R.—0.75 180°. L.—0.75 180°.	$\frac{4}{5}$ $\frac{4}{5}$
118	J. S.	M., 26.	L.—1.25 180°. R.+0.5 90° ◯ —2.25 180°.	$\frac{4}{4}$ $\frac{4}{5}$
119	W. B.	M., 29.	L.+1.5 90°. R. Em.	$\frac{4}{9}$
120	A. D.	F., 50.	L.+0.75 90°. R.+0.75 90°.	$\frac{4}{6}$ $\frac{4}{6}$
121	L. M.	M., 24.	L.—1.5 180° ◯ +0.75 90°. R.—1.5 15°.	$\frac{4}{6}$ $\frac{4}{6}$
122	C. W.	M., 11.	L.+0.5 20°. R. Em.	$\frac{4}{6}$
123	S. E. C.	F., 23.	L.+0.5 100°. R.+0.25 90°.	$\frac{4}{6}$ $\frac{4}{6}$
124	M. G.	F., 14.	L.—0.75 90°. R.+0.5.	$\frac{4}{6}$ $\frac{4}{5}$
125	A. J. F.	M., 33.	L.—0.5 90°. R.—0.5 90°.	$\frac{4}{6}$ $\frac{4}{6}$
126	M. E. T.	F., 22.	L.+0.75 90°. R.+6.5 90°.	$\frac{4}{4}$ $\frac{4}{4}$
127	A. C.	F., 18.	L.—1. R.—4 ◯ —3 180°.	$\frac{4}{9}$ $\frac{4}{18}$
128	S. C.	M., 42.	L —3 ◯ —1 90°. R.—4 ◯ —0.5 90°.	$\frac{4}{9}$ $\frac{4}{9}$
129	T. W. H.	M., 37.	L.+2.25 115°. R.+0.75 120°.	$\frac{4}{9}$ $\frac{4}{9}$
130	E. S.	M., 61.	L.+1.5 180°. R. Em.	$\frac{4}{8}$ $\frac{4}{12}$
131	F. D.	F., 21.	R.—0.5 180°. L. Em.	$\frac{4}{4}$
132	A. G.	F., 18.	L.—3 ◯ —0.75 180°. R.—4.	$\frac{4}{4}$ $\frac{4}{4}$

Number.	Name.	Sex and Age.	Correction Glasses and direction of axis of cylinder.	Vision after correction.
133	I. T.	F., 13.	L.—3 ◯—0.75 180°. R.—3 ◯—0.75 180°.	4/18 4/12
134	M. B.	F., 8.	L.+2.5 ◯+2.25 90°. R.+2 ◯+2.5 90°.	4/18 4/18
135	J. S	F., 22.	L.—1.25 180°. R.—3.5 180°.	4/6 4/18
136	J. C.	F. 32.	L.—0.5 180°. R.—0.75 180°.	4/6 4/6
137	M. P. B.	M., 23.	R.+4 ◯+3.5 90°. L.+4 ◯+3.5 90°.	4/1 4/18
138	W. M.	M., 21.	L.+3 ◯ +1.5 90°. R.+3 ◯ +1.5 90°.	4/6 4/6
139	P. G.	M., 18.	L.—1.25 180°. R.—0.75 180°	4/4 4/4
140	M T.	F.,40.	L.—10 ◯—1.5 180°. R.—15.	4/9 4/18
141	M. D.	F., 40.	L+1 90°. R+1.25 90°.	4/5 4/5
142	T. D.	M., 51.	L.—10 ◯—1.5 20°. R—10 ◯—1 110°.	4/12 4/12
143	A. H.	F , 48.	L.+0.75 ◯+0.5 150°. R.+0.75 ◯+0.5 40°.	4/6 4/6
144	F. H.	M., 14.	L.—1.5 40° ◯+2.75 120°+. R.—1 180° ◯+2.5 90°+.	4/18 ? 4/18
145	A. H. F.	M., 37.	L.+0.5 140°. R.+0 5 90°.	4/5 4/5
146	J. S.	F., 39.	L.+0 5 90°. R.+0.5 90°.	4/4 4/4
147	F. E.	M., 16.	L.—8 ◯—1.75 180°. R.—8.	4/12 4/12
148	D. W. P.	M., 38.	L.—4.5 ◯—0.5 40°. R.—4.5 ◯—1.5 180°.	4/5 4/5
149	E. P.	F., 13.	L. atrophy of o.n. R.—0.75 180°.	4/6
150	L. I'.	F., 16.	L.—0.75 45°. R.—0 75 135°.	4/6 4/6
151	R. I.	M., 26.	L.+4.5 90 (amblyopia). R.+1 ◯+0.75 75°.	4/9

Num-ber.	Name.	Sex and Age.	Correction glasses and direction of axis of cylinder.	Vision after correction.
152	J. J. M.	M., 22.	L.—0.75 180°. R,—0.75 180°.	4/4
153	R. M.	M., 23.	L. Fm R.+ .5 90°-	4/4
154	E. C.	F., 51.	L.—0.5 180°. R.—0.5 180°.	4/5 4/5
155	G. S. P.	M·, 45.	L.+1 75 R.—0.75 180°.	4/12 4/4
156	R. H.	M., 21.	L.—2 ◯ —1.5 90°. R.—2 ◯ —2.95°.	4/4 4/4
157	W. H. P.	M., 28.	L.—3 ◯ —3.5 5°. R.—2.5 ◯ —1 160°.	4/5 4/5
158	S. P. S.	F., 51.	L.+0.75 135°. R.+0.75 45°,	4/4 4/5
159	W. C. S.	M., 27.	L.—3 180°. R.—3 180°.	4/6 4/6
160	P. C. M.	M., 29.	L.+1. R.+0.5 90°.	4/4 4/4
161	E. S.	M., 27.	L.—4 ◯ —1 45°. R.—1 ◯ —1 180.	4/5 4/6
162	T. B.	M., 54.	L. ambly. R.+0.5 45°.	4/6
163	F. L.	F., 24.	L.+4 ◯ +3 180°. R.+2 ◯ +3 180.°	4/24 4/12
164	A. M.	F., 25.	L.+0 5 90°. R.+0.5 90°.	4/5 4/5
165	A. W. H.	M., 40.	L.+0.5 135°. R.+0.25 45°.	4/4 4/4
166	J. C. P.	M., 30.	L.+3. R.+2.25 ◯ +0.5 90°.	4/4 4/5
167	J. S.	M., 16.	L. amblyopic. R.+0.75 90°.	4/5
168	S. R.	F., 26.	L.+0.5 90°. R.+1. 90°.	4/4 4/4
169	S. C.	F., 14.	L —3.5 ◯ —0.75 90°. R.—3.5 ◯ —0.75 90°.	4/5 4/5
170	J. McE.	M., 21.	L.—4 10°. R.—3 ◯ —1. 170°.	4/12 4/6

9

Number.	Name.	Sex and Age.	Correction glasses and direction of axis of cylinder.	Vision after correction.
171	D. McE.	M., 21,	L.+1.25 ⊃ +3.5 50°. R.+2 ⊃ +4 105°.	4/6 4/12
172	L. M.	M., 50.	L.—6 ⊃ —2.5 90° R.—7 ⊃ —3 180°.	4/18 4/18
173	T. C. S.	M., 46.	L.—0.75 160°. R. Em.	4/4 4/4
174	E. T	M., 48.	L.+0 75 90°. R. Em.	4/9 4/9
175	L. P.	F., 30.	L.+0.5 110° ⊃ —0.75 20°. R.—1.25 160°.	4/5 4/5
176	F. G. D.	F., 38.	L.+0.75 70°. R.+0.75 110°.	4/5 4/5
177	M. M.	F., 16.	L.+1.75 90°. R.+1. 180°.	4/18 4/4
178	A. A. F.	F., 15.	L.+3. R.+0 75 90°.	4/9 4/6
179	A. B. S.	M., 45.	L.—1 ⊃ —2.5 180°. R.—0.75 180°.	4/12 4/12
180	S. A.	F., 42.	L.+5.5 135°. R.+1.75 45°.	4/12 4/12
181	E. N. W.	M., 29.	L.—9 ⊃ —1.25 180°. R.—9 ⊃ —1.25 180°.	4/6 4/6
182	A. W.	F., 9.	L.+5.5 180°. R.+5 ⊃ +3.5 90°.	4/36 4/18
183	J, B. M.	M., 25.	L.—0.75 180°. R.—0.75 180°.	4/4 4/4
184	M. C.	F., 70.	L.—1 180°. R. E. (Com. Cat.)	4/18 4/18
185	A. P.	F., 31.	L.—1 ⊃ +1 160°. R.—3.5 ⊃ —2.5 180°.	4/6 4/6
186	M. G.	F., 23.	L.—2.25 ⊃ —1 90°. R.—3.5 ⊃ —1 90°.	4/5 4/5
187	M. W.	F., 39.	L.—1 180°. R.—0.75 180°.	4/5 4/5
188	E. S.	M., 26.	L.—0.5 180°. R.—0.5 180°.	4/4 4/4
189	C. C. B.	M., 38.	L.+7 ⊃ +1 90°. R.+6 ⊃ +1 90°.	4/60 4/36

Num-ber.	Name.	Sex and Age.	Correction glasses and direction of axis of cylinder.	Vision after Correction.
190	L. J. D.	F., 39.	L.+1.5 90°. R.+1.5 ⌒ +5 90°.	$\frac{4}{5}$ $\frac{4}{5}$
191	V. E. P.	F., 54.	L. Amblyopia. R.+1 ⌒ +0.75 180°.	$\frac{4}{5}$
192	R. G.	M., 21.	L. Atrophied. R.+0.5 180°.	$\frac{4}{4}$
193	L. J.	F., 12.	L.—0.73 180°. R.—0.5 180°.	$\frac{4}{4}$ $\frac{4}{4}$
194	S. S. W.	M., 26.	L.—2.25 ⌒ —0.5 90°. R.—2.25 ⌒ —0.5 90°.	$\frac{4}{4}$ $\frac{4}{4}$
195	J. R. M.	M., 33.	L.—0.5 10°. R.—0.5 100°.	$\frac{4}{4}$ $\frac{4}{4}$
196	C. T.	F., 40.	L.+1 115°. R.+0.75.	$\frac{4}{5}$ $\frac{4}{5}$
197	A. I. D.	M., 57.	L.+1 ⌒ +0.5 90°. R.+1 ⌒ +0.5 90°.	$\frac{4}{5}$ $\frac{4}{5}$
198	D. P. G.	M., 15.	L.+1 ⌒ +0.75 90°. R.+1 ⌒ +0.75 90°.	$\frac{4}{5}$ $\frac{4}{5}$
199	A. J. G.	F., 50.	L.—1.5 ⌒ —0.75 120°. R. Amblyopia.	$\frac{4}{5}$
200	M. J. S.	M., 63.	L.—5 ⌒ —1 90°. R.—5 ⌒ —1 90°.	$\frac{4}{12}$ $\frac{4}{12}$
201	J. H.	M., 43.	L.—0.75 45°. R.—9.	$\frac{4}{9}$ $\frac{4}{18}$
202	J. S. B.	M., 36.	L.—0.75 ⌒ —1.5 115°. R.—0.5 50°.	$\frac{4}{18}$ $\frac{4}{9}$
203	H. C.	F., 40.	L.—0.5 90°. R.—0.5 90°.	$\frac{4}{5}$ $\frac{4}{5}$
204	M. E. M.	F., 50.	L.—1 135°. R.—1.	$\frac{4}{12}$ $\frac{4}{12}$
205	R. D. K.	F., 40.	L.+1.25 ⌒ +1.5 110°. R. Em.	$\frac{4}{9}$ $\frac{4}{4}$
206	H. B. C.	M., 29.	L. —4 ⌒ —2.5 165°. R.—6 ⌒ —2 180°.	$\frac{4}{6}$ $\frac{4}{6}$
207	H. C. H.	M., 53.	L.+0.75 10°. R.+0.75 115°.	$\frac{4}{4}$ $\frac{4}{4}$
208	H. L.	M., 28.	L.—2 ⌒ —2 15°. R.—7^s ⌒ —0.5 180°.	$\frac{4}{6}$ $\frac{4}{9}$

Number.	Name.	Sex and Age.	Correction glasses and direction of axis of cylinder.	Vision after correction.
209	C. F. B.	M., 32.	L. Em. R.+0.5 170°.	4/4 4/4.
210	J. H. M.	M., 48.	L.—1 115°. R. Em. (Corn. Cat.)	4/12 4/12
211	S. C. McD.	M., 51.	L.+2 ⊂ +0.75 180°. R.+0.75 ⊂ +0.75 180°.	4/5 4/5
212	M. W. R.	F., 21.	L.—1 ⊂ —2 180°. R.—1.25 ⊂ —0.75 180°.	4/4 4/4
213	V. J.	F., 22.	L.—0.75 180°. R.—1.25 180°.	4/6 4/5
214	L. B. S.	M., 13.	L.—1 180°. R.—1 180°.	4/4 4/4
215	J. P. J.	M., 24.	L.—0.5 180°. R.—2.5.	4/4 4/9
216	J. P. H.	M., 43.	L.+0.5 90°. R.—0.5 180°.	4/4 4/4
217	A. D. R.	M., 45.	L.+0.5 45°. R.+0.5 135°.	4/4 4/4—
218	G. M. D.	M., 52.	L.—1.5'. R.—1 ⊂ —0.75 180°.	4/4 4/4
219	R. N. B.	M., 40.	L. emmetropia. R.—0.75 180°.	4/4 4/4
220	M. R.	F., 24.	L.—1.25 170° ⊂ +3.75 80°. R.—0.75 10° ⊂ +3.25 100°.	4/12 4/12
221	C. D.	F., 41.	L.—3.5 ⊂ —0.5 180°. R.—4 ⊂ —0.5 180°.	4/4 4/4
222	W. F.	M., 48.	L. Em. R.—0.5 90°.	4/4 4/4
223	C. C. G.	M., 51.	L.+0.75 90°. R.+0 75 90°.	4/4 4/4
224	W. H. B.	M., 13.	L.—0.75 90°. R.—amblyopic.	4/4
225	E. E. W.	F., 30.	L.—2.5 45°. R.—3 135°.	4/4 4/4
226	H. T.	M., 41.	L.—3.5 ⊂ —0.75°. R.—8°.	4/5 4/12
227	M. E. M.	F., 15.	L.—1 ⊂ +0.5 90°. R.+1 ⊂ +2 80°.	4/4 4/12

Num-ber.	Name.	Sex and Age.	Correction glasses and direction of axis of cylinder.	Vision after correction.
228	C. M. R.	M., 26.	L.—amaurotic. R.—8 ⌣ —1 180°.	$^4/_6$
229	C. T.	F., 30.	L.—0.75 180°. R.—0.75 180°.	$^4/_5$ $^4/_5$
230	F. S.	F., 12.	L.—0.5 180°. R.—0.5 180°.	$^4/_4$ $^4/_4$
231	B. C.	F., 12.	L.—2.5 180° ⌣ +4 90°. R.+4 90°.	$^4/_{12}$ $^4/_{12}$
232	A. H. W.	F., 18.	L.—45 ⌣ —1.5 85°. R.—45 ⌣ —2 90°.	$^4/_6$ $^4/_6$
233	N. G.	F., 16.	L.—2.5 ⌣ —0.5 80°. R.—0.5 90°.	$^4/_9$ $^4/_6$
234	J. H. McB.	M., 40.	L.—1 90°. R.—1.5 95°.	$^4/_9$ $^4/_6$
235	A. R.	F., 20.	L.—0.5 90°. R.—0.5 90°.	$^4/_4$ $^4/_4$
236	B. P.	F., 25.	L.—1 180°. R.—1 180°.	$^4/_4$ $^4/_4$
237	D. B.	F., 16.	L.—0.5 ⌣ —0.75 180°. R.—0.75 ⌣ —0.75 180°.	$^4/_5$ $^4/_5$
238	D. T. W.	M., 33.	L.—2.75 ⌣ —1.5 100°. R.—3.5 ⌣ —1 100°.	$^4/_5$ $^4/_5$
239	H. H.	M., 18.	L.+0.75 180°. R.+0.75 180°.	$^4/_6$ $^4/_6$
240	E. P. H.	M., 49.	L.—1 ⌣ —0.5 180°. R. amblyopic.	$^4/_5$
241	J. M. E.	F., 34.	L.+0.5 90°. R.+0.75 90°.	$^4/_9$ $^4/_9$
242	A. M. M.	M., 38.	L.—0.75 180°. R.—0.75 180°.	$^4/_4$ $^4/_4$
243	R. K.	M., 50.	L.+1 180°. R. amblyopic.	$^4/_6$
244	W. W.	F., 38.	L.—0.5 135°. R.—0.5 45°.	$^4/_4$ $^4/_4$
245	G. J.	M., 38.	L.—0.75 90°. R.—1.25 90°.	$^4/_5$ $^4/_5$
246	M. M.	F., 48.	L.—0.75 135° ⌣ +0.75 45°. R.—0.75 45° ⌣ +0.75 135°.	$^4/_6$ $^4/_6$

Num-ber.	Name.	Sex and Age.	Correction glasses and direction af axis of cylinder.	Vision after correction.
247	C. W. S.	F., 30.	L.—0.5 180°. R.—0.5 180°.	4/4 4/4
248	F. L. G.	F., 16.	L.—0.5 180°. R.—0.5 180°.	4/4 4/4
249	P. X. D.	M., 41.	L.+3.5 90°. R.+2.5 90°.	4/24 4/24
250	L. H.	F., 18.	L.—0.5 180°. R.—0.5 180°.	4/4 4/4
251	I. W. B.	M., 40.	L.—0.5 180°. R.—0.5 180°.	4/5 4/5
252	J. S. B.	F., 48.	L.+0 75 35°. R.+0.75°.	4/6 4/6
253	D. B.	F., 18.	L.—3 ◡ —0 5 180°. R.—3 ◡ —0 5 180°.	4/5 4/5
254	E. M.	F., 25.	L.+0.5 175°. R.+0.5 180°.	4/4 4/4
255	J. J. A.	M., 40	L.—0.5 40°. R.—Em.	4/6
256	E. S.	F., 21.	L. Em. R.—0.75 90°.	4/6
257	Γ. S.	F., 26.	L.—0.75 50°. R.—0.75 140°.	4/9 4/9
258	R. J. F.	F., 40.	L.+0.75 135°. R.+0.75 45°.	4/5 4/5
259	K. B. F.	F., 40.	L.—0.5 180°. R.—0.5 180°.	4/4 4/4
260	C. E. P.	F., 30.	L.—0.5 180°. R.—0.5 15°.	4/12 4/12
261	R. B.	M., 31.	L.—1 70°. R.—1 110°.	4/6 4/6
262	C. S.	F., 30.	L.—0 5 180°. R.—0.5 180°.	4/4 4/4
263	M. W.	F., 65.	L.—10°. R.—6 ◡ —1 90°.	4/18 4/18
264	E. L.	F., 18.	L.+0.75 90°. R.+0.5°.	4/4 4/4
265	H. E.	F., 18.	L.+0.75 90°. R.+0.75 90°.	4/5 4/5

Number.	Name.	Sex and Age.	Correction glasses and direction of axis of cylinder.	Vision after correction.
266	L. H.	F., 29.	L.—0.5 180°. R.—0.5 180°.	4/5 4/5
267	J. H.	M., 45.	L.—0.25 135°. R.—0.5 45°.	4/4 4/4
268	R. G.	F., 18.	L.—0.5ˢ. R.—0.5 180°.	4/4 4/4
269	I. H. H.	F., 34.	L.+1.75 ◯ +1 90°. R.+1.75 ◯ +1 90°.	4/5 4/5
270	L. B.	F., 21.	L.+0.5 95°. R.+0.5 85°.	4/4 4/4
271	W. R. W.	F., 25.	L.—12ˢ. R.—12 ◯ —1.	4/9 4/9
272	J. H. W.	M., 28.	L.—6 ◯ —1 90°. R.—8 ◯ —1 90°.	4/6 4/6
273	G. W.	M., 51.	L.+3 180°. R.+2 180°.	4/6 4/6
274	A. L.	F., 16.	L.—1.5 ◯ —0.5 45°. R.—1.5 ◯ —0.5 150°.	4/4 4/4
275	F. S. P.	M., 26.	L.+1 175°. ◯ —1.5°. R.+1 10° ◯ —1 170°.	4/5 4/5
276	J. S. B.	M., 50.	L.+1ˢ. R.+1 ◯ +0.5 180°.	4/4 4/▪
277	J. L.	F., 20.	L.—0.75 90°. R.—0.75 90°.	4/4 4/4
278	F. W. I.	M., 27.	L.+0.5 90°. R.+0.5 90°.	4/4 4/4
279	C. A. P.	M., 45.	L.+1 ◯ +0.5. R.+1 ◯ +0.5.	4/4 4/4
280	C. A. N.	F., 26.	L.—0.75 180°. R.—0.75 180°.	4/4 4/4
281	L. A.	F., 21.	L.+0.75 135°. R.+0.75 45°.	4/4 4/4
282	D. E. S.	F., 30.	L.—1.5 90°. R.—2.5 90°.	4/4 4/5
283	L. B.	F., 12.	L.—4. R.—3 ◯ —1.5 180°.	4/9 4/9
284	M. V.	F., 13.	L.+1 100°. R.+1 80°.	4/5 4/5

Number.	Name.	Sex and Age.	Correction glasses and direction of axis of cylinder.	Vision after correction.
285	M. L. S.	F., 40.	L.+1 80°. R.+0.75 95°.	4/4 4/4
286	S. C.	F., 36.	L.+0.75 90°. R.+0.75 90°.	4/4 4/4
287	W. F. R.	M., 22.	L.—0.5 180°. R.—0.5ˢ.	4/4 4/4
288	E. M.	F., 50	L.+0.75 180°. R.+1 ⊃ +0.75 180°.	4/6 4/6
289	E. P.	F., 30.	L.+2.5 90°. R.+2.5 90°.	4/6 4/6
290	J. P. M.	M., 53.	L.—10 ⊃ —1 175°. R.—10 ⊃ —1 10°.	4/9 4/12
291	E S. P.	M., 40.	L.—0.75 180°. R.—0.75 180°.	4/5 4/5
292	C. W. F.	F., 30.	L —0.75 180°. R.—0 75 180°.	4/4 4/4
293	J. H. W.	F., 40.	L.+25 ⊃ +0 5 100°. R.+1 ⊃ +1 90°.	4/6 4/5
294	A. S.	F., 21.	L.+0.75 180°. R. Em.	4/4 4/4
295	J. O. S.	M., 48.	L. Em. R.+2 25 ⊃ +0.5 100°.	4/4 4/5
296	E. M. B.	F , 40.	L.+1.75 ⊃ +0.5 90°. R.+1.75 ⊃ +0.5 90°.	4/5 4/5
297	C. A. H.	M., 30.	L.—0.75 ⊃ —0.75 60°. R.—0.5 ⊃ —0.5 120°.	4/5 4/5
298	K. S.	F., 17.	L.—0.5 40°. R.—0.5 180°.	4/5 4/5
299	S. A.	F., 42.	L +0.75 100°. R +0.75 180°.	4/4 4/4
300	W. H. K.	M., 23.	L —3.5 ⊃ 1 —5 155°. R.—4.5ˢ.	4/6 4/6
301	E. W. N.	F., 40.	L.—1ˢ. R.—5 ⊃ —1 20°.	4/9 4/9
302	J. B. B.	M., 50.	L.+2.25 ⊃ +0.75 30°. R.+1.5 ⊃ +0.5 145°.	4/6 4/6
303	D. E. S.	M., 36.	L —1.5 180°. R.—1 170°.	4/4 4/4

Num- ber.	Name.	Sex and Age.	Correcting glasses and direction of axis of cylinder.	Vision after correction.
304	H. W. G.	F., 35.	L.—0.75 180°. R.—0.75 180°.	$\frac{4}{4}$ $\frac{4}{4}$
305	W. A. K.	M., 41.	L.—1 ◯ —2 180°. R.—1.5 180°.	$\frac{4}{9}$ $\frac{4}{9}$
306	T. B. M.	M., 23.	L.—0.5 90°. R.—0.75 100°.	$\frac{4}{5}$ $\frac{4}{6}$
307	R. R. W.	M.,33.	L. Amblyopic. R.—0.5 180°.	$\frac{4}{4}$
308	E. C.	F., 17.	L.+0.75 90°. R.—1.25 180°.	$\frac{4}{4}$ $\frac{4}{5}$
309	J. B. K.	M., 14.	L.—5 180°. R.—5 180°.	$\frac{4}{12}$ $\frac{4}{12}$
310	T, C.	M., 25.	L.—0.75 180°. R.—0.75 180°.	$\frac{4}{5}$ $\frac{4}{4}$
311	J. F. K.	M., 43.	L.—0.75;180°. R. Em.	$\frac{4}{4}$
312	A. M.	F., 21.	L. Em. R.—0.75 180°.	$\frac{4}{4}$ $\frac{4}{4}$
313	M. L.	F., 21.	L.—2 ◯ —2.5 130°. R.—4 ◯ —1.5 130°.	$\frac{4}{18}$ $\frac{4}{24}$
314	S. M. B.	M., 40.	L.+2.5 70°. R. Em.	$\frac{4}{18}$ $\frac{4}{4}$
315	K. A.	F., 12.	L—4 10°. R.—4 180°.	$\frac{4}{18}$ $\frac{4}{18}$
316	L. J. B.	M.,45.	L.+3.5 ◯ +1.25 160°. R.+4 ◯ +1 180°.	$\frac{4}{9}$ $\frac{4}{12}$
317	H. C. L.	F., 33.	L +2 ◯ +1 165°. R.+2 ◯ +1 15°.	$\frac{4}{5}$ $\frac{4}{5}$
318	W. B. H.	M., 23.	L.—0.5 180°. R.—0.5 180°.	$\frac{4}{4}$ $\frac{4}{4}$
319	C. N.	F., 22.	L.—2 180°. R.—2 180°.	$\frac{4}{5}$ $\frac{4}{5}$
320	H. F.	M., 33.	L.—0.5 20°. R.—0.5 155°.	$\frac{4}{4}$ $\frac{4}{4}$
321	S. P. T.	F., 40.	L.—4 140°. R.—4 165°.	$\frac{4}{6}$ $\frac{4}{24}$
322	M. W.	F., 18.	L.+6 105°. R.+2 180°.	$\frac{4}{36}$ $\frac{4}{9}$

Num-ber.	Name.	Sex and Age.	Correcting glasses and direction of axis of cylinder.	Vision after correction.
323	S. R. B.	F., 40.	L.—0.75. R.—0.5 100°.	$\frac{4}{6}$ $\frac{4}{6}$
324	F. O.	F., 18.	L.—0.5 35°. R.+0.5 30°.	$\frac{4}{6}$ $\frac{4}{4}$
325	A. P.	F., 55.	L.+3. R.+1.75 ⊃ +0.75 90°.	$\frac{4}{9}$ $\frac{4}{9}$
326	M. L. F.	F., 21.	L.+0.5 180°. R.+0.75 180°.	$\frac{4}{5}$ $\frac{4}{5}$
327	J. C.	F.. 17.	L.+1.25 120°. R.+0.75 90°.	$\frac{4}{4}$ $\frac{4}{4}$
328	D. M. O.	M., 38.	L.—0.5 90°. R.—0.25 90°.	$\frac{4}{4}$ $\frac{4}{4}$
329	N. H.	F., 13.	L. Em. R.—2.25 180°.	$\frac{4}{5}$ $\frac{4}{12}$
330	A. B.	F., 30.	L.—0.75 30°. R.—2.5.	$\frac{4}{9}$ $\frac{4}{9}$
331	M. B.	F., 14.	L.—3.5 ⊃ —0.5 90°. R.—3.5.	$\frac{4}{4}$ $\frac{4}{4}$
332	M. D.	F., 13.	L.+10 ⊃ +0.75 180°. R.+10 ⊃ +0.75 180°.	$\frac{4}{9}$ $\frac{4}{9}$
333	J. J. J.	M., 47.	L.+1. R.+2.5 ⊃ +1 120°.	$\frac{4}{5}$ $\frac{4}{6}$
334	B. A. P.	F., 45.	L.—12. R.—8 ⊃ —1 145°.	$\frac{4}{12}$ $\frac{4}{4}$
335	J. M. K.	M., 26.	L.+0.75 105°. R.—2.	$\frac{4}{4}$ $\frac{4}{4}$
336	R. J. M.	M., 32.	L.—6 ⊃ —1 180°. R.—2.	$\frac{4}{4}$ $\frac{4}{5}$
337	W. F. M.	M., 47.	L.+0.5 90°. R.+0.5 90°.	$\frac{4}{4}$ $\frac{4}{4}$
338	D. B. S.	F., 45.	L.—0.5 180°. R.—0.75 135°.	$\frac{4}{4}$ $\frac{4}{4}$
339	A. T.	F., 19.	L. Em. R.+0.5 115°.	$\frac{4}{4}$ $\frac{4}{4}$
340	L. S. B.	M., 23.	L.—1.75 170°. R.—1.25.	$\frac{4}{4}$ $\frac{4}{4}$
341	M. F.	F., 50.	L.+3 180° R.+2.25 ⊃ +0.75 180°.	$\frac{4}{60}$ $\frac{4}{6}$

Number.	Name.	Sex and Age.	Correction glasses and direction of axis of cylinder.	Vision after correction.
342	H. B. B.	F.,46.	L.+1.5 ⊃ +0.5 90°. R. Amblyopic.	4/4
343	L. N.	F., 20.	L.—10 ⊃ —0.5 180°. R.—9 ⊃ —0.5 180°.	4/6 4/9
344	E. R. B.	F., 40.	L.+1.5 ⊃ +0.5 90°. R.+1. ⊃ +0.5 90°.	4/4 4/4
345	A. W.	F., 19.	L.—0.75 180°. R.—Em.	4/4 4/4
346	E. B.	F., 28.	L.—1.5 135°. R.—1.5 ⊃ —1.5 45°.	4/5 4/5
347	D. S.	M., 33.	L.+2.25 ⊃ +1.5 120°. R.+2.5.	4/12 4/9
348	S. C.	F.. 38.	L.—4. ⊃ —0.75 180°. R.—3.5 ⊃ —0.5 180°.	4/5 4/5
349	E. F.	F., 39.	L.+2.5 ⊃ +1 170°. R.+2.5 ⊃ +1 180°.	4/18 4/18
350	G. C. K.	F., 50.	L.+4 45°. R+0.5 ⊃ +1 90°.	4/18 4/5
351	D. C.	F., 41.	L.—1.5 180°. R.—2 180°.	4/5 4/5
352	C. H. M.	M., 40.	L.—0.5 20°. R.—0.5 140°.	4/5 4/5
353	M. A.	F., 18.	L.—0.5 ⊃ —0.75 180°. R.—1.5 ⊃ —0.75 180°.	4/4 4/4
354	M. W. R.	F., 40.	L.+2 140°. R.+1 10° ⊃ —1.5 100°.	4/5 4/6
355	E. M.	F., 48.	L.—2 10° ⊃ +4.75 100°. R—2 170 ⊃ +5 80°.	4/18 4/9
356	N. M.	F., 50.	L.—0.5 90°. R.—4 ⊃ —0.5 90°.	4/5 4/9
357	A. T. B.	M., 51.	L.—4 ⊃ —1 70°. R.—4 ⊃ —1 130°.	4/4 4/4
358	M. A.	F., 45.	L—0.5 90°. R.—4.	4/4 4/12
359	M. W.	F., 50.	L.—1 ⊃ —0.75 60°. R.—1 ⊃ —0.75 120°.	4/4 4/4
360	B. C.	F., 36.	L.+2 ⊃ +0.5 90°. R. +0.75 90°.	4/4 4/4

Number.	Name.	Sex and Age.	Correction Glasses and direction of axis of cylinder.	Vision after correction.
361	E. T. P.	M., 48.	L. Amblyopic. R.+0.5 90°.	4/60 4/4
362	E. J.	F., 50.	L.+2 ◠ +0 5 5°. R.+2 ◠ +0.5 175°.	4/4 4/4
363	L. P.	F., 38.	L.+2.5 ◠ +0 5 180°. R.+2.5 ◠ +0.5 180°.	4/4 4/4
364	A. D. J.	M., 43.	L.+1. R.+0.5 ◠ +0.5 135°.	4/4 4/4
365	A. W.	M., 15.	L.—0 75 ◠ —0.5 90°. R.—1 ◠ —0.5 90°.	4/4 4/5
366	A. McG.	F., 17.	L.—0.5 180°. R.—0.5 180°.	4/4 4/4
367	J. S. H.	M., 32.	L.+0.5 90°. R.+0.5 90°.	4/4 4/4
368	M. E. G.	F., 33.	L —4 ◠ 1.5— 180°. R.—4.	4/5 4/5
369	J. M. C.	F., 18.	L.—0.5 100°. R.—2.25.	4/4 4/4
370	N. H.	F., 26.	L.—1. R.—2.5 ◠ —2 90°.	4/4 4/5
371	P. M.	F., 18.	L.+0.5 90°. R.+0.5 90°.	4/4 4/4
372	M. L. N.	F., 31.	L—1 45°. R+0.5 90°.	4/4 4/4
373	H. T. H.	F., 40.	L. Em. R.+1 90°.	4/4 4/4
374	J. L. J.	M., 46.	L.+0.75 135°. R.+0.75.	4/5 4/5
375	W. D.	F., 24.	L. Em. R.—0.5 90°.	4/4 4/4
376	S. B. M.	F., 27.	L.—0.5 180°. R.—0.5 180°.	4/4 4/4
377	C. A. R.	M., 27.	R.+4 180° ◠ —3 90°. R —6 90° (keratoconus).	4/12 4/9
378	N. R.	F., 14.	L.+0 75 180°. R.+0.75 180°.	4/4 4/4
379	J. K.	M., 70.	L.+0.5 180°. R. Em.	4/6

Number.	Name.	Sex and Age.	Correction glasses and direction of axis of cylinder.	Vision after correction.
380	M. V.	F., 16.	L.+4.5 95°. R—1 175° ⊃ +2.5 85°.	$\frac{4}{6}$ $\frac{4}{6}$
381	A. D.	F., 14.	L.—4. R.—0.5 170°.	$\frac{4}{9}$ $\frac{4}{4}$
382	A. S.	M., 42.	L.+0.75 90°. R.+0.5 90°.	$\frac{4}{4}$ $\frac{4}{4}$
383	S. W. C.	F. 33.	L.+6 ⊃ +3 135°. R.+2 5 ⊃ +4.5 90°.	$\frac{4}{36}$ $\frac{4}{12}$
384	H. W. H.	M., 26.	L.—0.5 180°. R.—0.5 180°.	$\frac{4}{4}$ $\frac{4}{4}$
385	J. W.	F., 50.	L +1.25 100°. R +1.25 170°.	$\frac{4}{4}$ $\frac{4}{4}$
386	A. H.	M., 44.	L.—0.5 45°. R.—0.75 135°.	$\frac{4}{4}$ $\frac{4}{4}$
387	E. L.	F., 18.	L.—2. R.—0.5 ⊃ —5 180°.	$\frac{4}{4}$ $\frac{4}{4}$
388	L. J.	M., 29.	L.—0 5 80°. R.—0.5 100°.	$\frac{4}{4}$ $\frac{4}{4}$
389	W. M. G.	F., 50.	L.+0 75 ⊃ +4.5 90°. R.+0.75 ⊃ —3.5 90°.	$\frac{4}{5}$ $\frac{4}{5}$
390	L. H. C.	F., 36.	L.+3 90°. R.—3 150°.	$\frac{4}{5}$ $\frac{4}{5}$
391	S. F.	F., 50.	L.+1. R.+0.5 ⊃ +0.75°.	$\frac{4}{5}$ $\frac{4}{5}$
392	L. G.	F , 18.	L.+1. R.+1 ⊃ +0.5 90°.	$\frac{4}{4}$ $\frac{4}{4}$
393	E. L.	M., 19.	L. Em. R.—1.5 ⊃ —0.5 90°.	$\frac{4}{4}$ $\frac{4}{4}$
394	J. B. M.	F., 38.	L. Em. R.—1.5 ⊃ —1.25 180°.	$\frac{4}{18}$
395	S. C.	F., 17.	L.+0.75 90°. R. Em.	$\frac{4}{4}$ $\frac{4}{4}$
396	H. W. S.	F., 28.	L.+1.25 90°. R.+4.5 90°.	$\frac{4}{5}$ $\frac{4}{18}$
397	A. M.	F., 21.	L.+4 90°. R.+1 ⊃ +1.5 90°.	$\frac{4}{60}$ $\frac{4}{5}$
398	A. O'C.	F., 32.	L.+4 ⊃+2 90°. R.+3 5 ⊃+2 90°.	$\frac{4}{4}$ $\frac{4}{4}$

Number.	Name.	Sex and Age.	Correcting glasses and direction of axis of cylinder.	Vision after correction.
399	K. H.	F., 50.	L.—1.5 15°. R.—1 180°.	$\frac{4}{4}$ $\frac{4}{4}$
400	H. H. N.	M., 48.	L.+1.5 90°. R.+1 90°.	$\frac{4}{5}$ $\frac{4}{4}$
401	I. R.	F., 16.	L.—1 180°. R,—1 180°.	$\frac{4}{4}$ $\frac{4}{4}$
402	H. M. I.	F., 33.	L.—1 ⊃ —0.5 180°. R.—0.5 180°.	$\frac{4}{4}$ $\frac{4}{4}$
403	O. I.	M., 47.	L.+0.5 90°. R.+0.5 90°.	$\frac{4}{4}$ $\frac{4}{4}$
404	H. K.	F., 36.	L.+3.5 105°. L.+4.5 70°.	$\frac{4}{18}$ $\frac{4}{18}$
405	C. B.	F , 34.	L.+5.5 90°. R.+3 90°.	$\frac{4}{18}$ $\frac{4}{18}$
406	T. K. S.	M., 47.	L.+0.5 90°. R.+6 100°.	$\frac{4}{4}$ $\frac{4}{24}$
407	A. N.	F., 19.	L.—4.5 ⊃ —1.25 180°. R.—4.5 ⊃ —0.5 190°.	$\frac{4}{4}$ $\frac{4}{4}$
408	G. M. W.	M , 26.	L.—0.75 90°. R.—0.5 90°.	$\frac{4}{4}$ $\frac{4}{4}$
409	F. B. N.	M., 28.	L.—0.75 90°. R.—0.75 90°.	$\frac{4}{5}$ $\frac{4}{5}$
410	S. F.	F., 29.	L.+1 ⊃ +4.5 90°. R.+5 90°.	$\frac{4}{9}$ $\frac{4}{9}$
411	G. M.	M., 36.	L.—3 ⊃ —3.5 45°. R.—4 ⊃ —2 90°.	$\frac{4}{24}$ $\frac{4}{24}$
412	L. H.	F., 27.	L.—0.75 180°. R.—0.75 180°.	$\frac{4}{5}$ $\frac{4}{5}$
413	L. B.	F., 10.	L.+2.5 95°. R.+3 95°.	$\frac{4}{5}$ $\frac{4}{9}$
414	M. I.	F., 21.	L.—0.5 180°. R.—0.25 180°.	$\frac{4}{4}$ $\frac{4}{4}$
415	N. P. S.	F., 23.	L.+1 ⊃ +0.5 90°. R.+1 ⊃ +0.75 90°.	$\frac{4}{4}$ $\frac{4}{4}$
416	H. A. C.	F., 50.	L.—1 ⊃ —0.5 45°. R.—0.5 90°.	$\frac{4}{5}$ $\frac{4}{5}$
417	C. S. J.	M., 35.	L.+2 ⊃ +1 90°. R.+2 ⊃ +1 45°.	$\frac{4}{4}$ $\frac{4}{4}$

Number.	Name.	Sex and Age.	Correction glasses and direction of axis of cylinder.	Vision after correction.
418	C. M.	F., 21.	L.—1 180°. R.—1 180°	$\frac{4}{4}$ $\frac{4}{4}$
419	K. H.	F., 20.	L.—4 ⌒ —1.25 30°. R.—4 ⌒ —0.75 150°.	$\frac{4}{4}$ $\frac{4}{4}$
420	M. F.	F., 56.	L.+2 ⌒ +1 80°. R.+2 ⌒ +1 90°.	$\frac{4}{5}$ $\frac{4}{5}$
421	K. B.	F., 50.	L.+2 ⌒ +2.5 100°. R.+1.5 ⌒ +1.5 100°.	$\frac{4}{4}$ $\frac{4}{4}$
422	M. T.	F., 27.	L.+0.5 90°. R.+0.5 90°.	$\frac{4}{4}$ $\frac{4}{4}$
423	C. M.	F., 40.	L.+0.5 115°. R. E.	$\frac{4}{4}$ $\frac{4}{4}$
424	L. T. L.	F., 30.	L.—3 ⌒ —0.75 90°. R.—3 ⌒ —0.75 90°.	$\frac{4}{5}$ $\frac{4}{5}$
425	A. G. R.	M., 23.	L.—2.5 ⌒ —0.75 10°. R.—3 5 ⌒ —0.75 170°.	$\frac{4}{5}$ $\frac{4}{5}$
426	E. G.	F., 38.	L. E. R.—0.75 180°.	$\frac{4}{4}$ $\frac{4}{4}$
427	E. F.	F., 50.	L.—0.5 100°. R.—0.5 80.	$\frac{4}{4}$ $\frac{4}{4}$
428	W. H. S.	M., 17.	L.—0.5 180°. R. E.	$\frac{4}{6}$ $\frac{4}{4}$
429	J. L. M.	F., 48.	L. E. R.—1.5 175°.	$\frac{4}{4}$ $\frac{4}{5}$
430	M. L. T.	F., 35.	L.—1.5 180°. R.—2.25 180°.	$\frac{4}{4}$ $\frac{4}{5}$
431	L. T.	F., 40.	L.—1.5 135°. R.—0.75 45°.	$\frac{4}{5}$ $\frac{4}{4}$
432	M. E. O.	F.. 48.	R.—0.75 90°. R. E.	$\frac{4}{4}$ $\frac{4}{4}$
433	C. C.	F., 21.	L.+1.5 110°. R.—0.5 180°.	$\frac{4}{5}$ $\frac{4}{4}$
434	A. D. H.	M., 44.	L.—0.5 100°. R.—0.5 80°.	$\frac{4}{6}$ $\frac{4}{6}$
435	F. R. G.	M., 25.	L.—0.75 ⌒ —0.75 75°. R.— ⌒ —0.75 105°.	$\frac{4}{5}$ $\frac{4}{5}$
436	L. A.	F., 42.	L.+0.5 80°. R.+0.5 100°.	$\frac{4}{4}$ $\frac{4}{4}$

Number.	Name.	Sex and Age.	Correction glasses and direction of axis of cylinder.	Vision after correction.
437	N. A.	F., 31.	L.—5. R.—4.5 ⊃ —0.5 180°.	4/6 4/6
438	T. M.	F., 53.	L.+3 ⊃ +1 50°. R.+ 1.5⊃ +1.5 90°.	4/12 4/4
439	C. A. C.	F., 50.	L.+1.5 ⊃ +0.5 130°. R.+2.25.	4/5 4/5
440	A. J.	F., 36.	L.+3.5 ⊃ +0.75 90°. R.+3.5 ⊃ +0.75 90°.	4/4 4/4
441	A. R.	F., 42.	L.+1. R.+3 ⊃ +2 90°.	4/4 4/36
442	G. R. G.	F., 33.	L.—0.5 20°. R.—1 155°.	4/4 4/5
443	W. D. J.	M., 25.	L.+0.5 90°. R.+0.5 90°.	4/4 4/4
444	C. E. R.	M., 40.	L.+0.5 90°. R.+05. 90°.	4/4 4/4
445	M. L.	M., 21.	L.—3 ⊃ —0.5 90°. R.—3 ⊃ —0.5 90°.	4/4 4/4
446	N. B.	F., 18.	L.—0.5 180°. R.—1 180°.	4/4 4/4
447	M. B.	F., 19.	L.—0.75 160°. R.—0.5 10°.	4/4 4/4
448	L. S.	F., 17.	L.—2 ⊃ —0.5 35°. R.—1 ⊃ —0.5 180°.	4/4 4/4
449	V. H.	M., 17.	L.—3 ⊙ —0.5 55°. R.—3.	4/5 4/5
450	J. T. K.	M., 21.	L.—7 ⊃ —0.75 180°. R.—7.	4/4 4/4
451	M. C. T.	M., 48.	L.—6 ⊃ —1 60°. R.—4 ⊃ —0.5 75°.	4/4 4/4
452	B. B.	F., 17.	L.—1.5 ⊃ —0.75 180°. R.—1.5 ⊃ —0.75 180°.	4/4 4/4
453	B. S.	M., 19.	L.—1 ⊃ —0.5 90°. R.—1 ⊃ —0.5 90°.	4/4 4/4
454	S. C.	M., 21.	L —0.5 180°. R.—0.5 180°.	4/4 4/4
455	J. H.	M., 17.	L.+0.5 90°. R.+2.	4/4 4/12

Number.	Name.	Sex and Age.	Correction glasses and direction of axis of cylinder.	Vision after correction.
456	J. M.	M., 46.	L.+180° ⌣ −2 170°. R.+1 90° ⌣ −2 180°.	4/5 4/5
457	M. D.	F., 15.	L.+0.5 90°. R.+0.5 90°.	4/4 4/4
458	G. S.	F., 28.	L.+2.25 ⌣ +0.5 90°. R.−2.25 ⌣ +0.5 90°.	4/4 4/4
459	H. C. F.	F., 50.	L.+0.5 180°. R.+0.5 180°.	4/4 4/4
460	A. S.	M., 71,	L.+1 ⌣ +0.5 90°. R.—Amblyopic.	4/5
461	J. B	M., 15.	L. E. R.+0.5 110°.	4/4 4/5
462	M. F.	F., 30.	L+1 ⌣ +1 90°. R.+1.5.	4/4 4/4
463	A. A.	F., 20.	L.−2.5 ⌣ −0.5 100°. R.−2.5 ⌣ −0.5 100°.	4/4 4/4
464	P. J. M.	M., 40.	L.+4 ⌣ +1 180°. R.+0.5 180°.	4/12 4/4
465	E. C.	F., 41.	L.+1 ⌣ +2.5 90°. R.+1 ⌣ +2.5 90°.	4/5 4/5
466	M. S.	F., 25.	L.−0.75 10°. R.−0.75 170°.	4/5 4/5
467	Wm. S.	M., 33.	L.−1.25 180°. R.−0.5 180°	4/4 4/4
468	G. P.	F., 18.	L.+1 170°. R.−1.	4/12 4/12
469	C. C. J.	F., 26.	L.+1 90° ⌣ −3 180°. R.+1.5 90° ⌣ −4 180°.	4/9 4/9
470	G. W. K.	M., 57.	L.+0.5 45°. R.+0.75 135°.	4/4 4/4
471	J. W. H.	F., 40.	L.+1 ⌣ +1 90°. R. E.	4/4 4/4
472	A. S.	F., 36.	L.+0.5 135°. R. E.	4/4 4/4
473	H. S	F., 13.	L— 0.5 180°. R —0.5 180°.	4/4 4/4

ADDENDA AND CORRIGENDA.

Beginning at the ninth line from the top, page 34, for "one of these represents, etc.," read: "The first of these tables shows the focus of a lens of 100 inches focus for every 5° of rotation on its vertical axis, in the vertical meridian. The second table shows the focus in the horizontal meridian." In the table, for "horizontal inclination," read "vertical meridian," and for "vertical inclination" read "horizontal meridian."

Page 36.—9th line from top, for "$-\frac{1}{70}$" read "$+\frac{1}{70}$."

Page 41.—The "3" at the bottom and to the left of the diagram should be "—3."

Page 74.—18th line from the top, for "hydrochlorate" read "hydrobromate."

Page 96.—8th line from top, insert "horizontal" before "meridian."

Page 131.—2nd line from top, for "larger" read "longer."

Page 162—25th line from top, insert "right" before "angles."

Appendix.—1st and 4th lines for "475" read "473."

Appendix.—4th line from the top, for "41 per cent." read "15 per cent."

The test-types and fan of Snellen used for diagnostic purposes can be obtained from Meyrowitz Bros., New York; Queen & Co., Philadelphia, or other prominent opticians.

The refraction ophthalmoscope with attachment for cylinders described on page 99 *et seq.* may be had of James W. Queen & Co., Philadelphia, who can also attach a clip for holding the cylinders to most of the refraction-ophthalmoscopes in common use.

Men Oper page 32

INDEX.

A

D

E

Y

Z

TABLES.